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RESEARCH ARTICLE

Advancing STEM education and research through preparing students with special interest in mathematics and science

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Abstract: This paper reports on the teaching of integrated STEM disciplines at the officially best school for creative and talented students of the Russian Federation. The paper shares how the success of this integration and the advancement of STEM education and research within the school is due to historical, cultural, and national practices of fostering creativity and giftedness at the pre-college level. Signature pedagogy of using concrete problems as a motivation for the study of abstract ideas is discussed. The merit of using traditional skills in mathematics in the digital era is demonstrated in the integrated context of entrance examination to the school and the modern-day digital technology. Several examples of research-oriented projects completed by the students at the school are presented.

Keywords: giftedness, competitions, STEM, robotics, professional development

1 Introduction

This paper reflects on teaching integrated STEM (science, technology, engineering, mathematics) disciplines in a school for creative and talented students (starting from age 11) with special interest in mathematics and science. Since its inception in 1961, the school enjoyed having among its students 78 Gold Medalists of International Olympiads in such multidisciplinary subjects as mathematics, physics, informatics, robotics, astronomy, and astrophysics. One graduate of the school was elected as full member of Russian Academy of Sciences in the division of mathematics (the highest national recognition of scientific achievements) and (to the best of the authors' knowledge) at least fourteen graduates of the school became corresponding members of the Academy (the second highest national recognition) in different STEM-related disciplines. At the international level, two of the school graduates became Fields medalists [1], an award often referred to as the Nobel Prize for mathematicians under the age of 40.

The paper is co-authored by a former mathematics teacher at the school in the late 1980s and its current Principal who was a student there at that time. It aims to provide an insight, through different student-teacher-administrator lenses, about didactic approaches to talent development and expansion of creativity within a STEM-rich integrative learning milieu. A few examples of creative work by students and teachers of the school (referred to below as Lyceum) will be delineated.

The authors will describe the above-mentioned pedagogical successes in teaching STEM subjects using the theoretical framework of the 5 E's – **enabling**, **engaging**, **empowering**, **enriching**, and **encouraging**. This framework was introduced in Freiman et al. (2018) [2] in the context of developing creativity through technology-enhanced mathematics education. The framework can be extended to other STEM disciplines. To this end, different unique features of the Lyceum, its students, and faculty will be highlighted in the paper.

The paper argues that the presence in the Lyceum of educationally unique laboratories of robotics, microelectronics, chemistry, biology, physical optics, nanotechnology, and computer music **enables** multiple talents not only to be duly treated but, better still, utilized and motivated in creative ways under the umbrella of application of advanced mathematics and science curricula to solving real-life problems. As mentioned in Wigfield et al. (2012) [3] with reference to Stipek (1996) [4], in order for a subject matter to be motivational, the curriculum should be challenging for students, regardless of their level of giftedness. Furthermore, teachers' appropriate responses to students' success in overcoming the challenges are important means of maintaining their intrinsic motivation to learn. More recently, Sternberg ([5], p. 104) argued that one of the critical issues in the education of gifted "is to accelerate and at the same time enrich their learning so that students are constantly challenged and, hopefully, never bored". Both acceleration and enrichment of learning in the school are the responsibilities of teachers and administrators who realize the importance of allowing students to utilize all possible advances of the technological society. This position bears on to a newly created Center for Digital Education at the Lyceum in which mathematical computations serve as building blocks of epistemically rich approaches to a variety of laboratory experiments "suggested by the world of external phenomena" ([6], p. 440), **enabling** significant educative outcomes of the approaches and, by implication, **empowering** teachers with the sense of self-efficacy and success in teaching.

The paper shares the initiative of combining curricula of mathematics and science courses with individual and/or group work in the above-mentioned laboratories as an extension of administrative flexibility principle [7] used at forward-looking research universities to the learning environment at the middle and secondary levels. Following this principle, the administration provides the Lyceum with **engaging** education that integrates theory and practice of STEM disciplines and ultimately leads to students' modest yet talented contributions to industries with creative ideas of improving already existing real-life projects. Put another way, self-directed learning [8] by gifted students "allows them not only to work within a learning environment, but also to create that environment" ([5], p. 104).

In mathematics, students create such environment through posing new problems for themselves [9], looking for multiple ways of solving a problem [10–12], going beyond the information given [13] and generating ideas "that the user may or may not develop ... [becoming evident after] the developments will be tried out, validated or rejected ... following the diversity of the situations and projects the users set for themselves" ([14], p. 186). In particular, the Lyceum's Center for the Development of Open Online Courses, an analogue of MOOCs (massive open online courses) movement in tertiary education [15], **enriches** pre-college STEM education by making available discipline-specific instructional practices, developed by teachers at the Lyceum (see, for example, Pratusevich et al. (2019) [16] – one of the most popular Russian textbooks for advanced study of pre-college algebra and calculus), to educators interested in teaching for creativity and to students eager to learn how to be creative outside of practicing pretty standard exam-oriented algorithms that are mandatory for all schools in the country, regardless of their status. Collectively, the teachers of the Lyceum published some 30 textbooks and 60 peer-reviewed articles. The Lyceum employs one teacher with a habilitation degree and eleven teachers with PhD degrees in STEM-related disciplines.

Another issue that the paper shares is how various captivating achievements of older students on the spectrum of local to international competitive successes **encourage** younger students to nurture their emerging talents by participating in different after school and summer extracurricular activities [17]. Sponsored by leading corporations and experienced faculty members from top universities, the latter activities **empower** talented students from other parts of the country to become enrolled in the boarding extension of the Lyceum that enrolls 77 students from 35 regions of Russia, Ukraine, and Belorussia. This extension initiative is the testament of educational equity and democratic access to quality STEM education provided by the administration of the Lyceum.

All things considered, using the 5 E's theoretical framework, the paper's goal is to contribute to more coherent understanding of the role of giftedness and creativity in advancing STEM integration and professional development of teachers of mathematics and science at the middle and secondary school levels. It has been a long cultural tradition in St Petersburg for notable mathematicians, physicists, and other members of the STEM scientific community to participate in a variety of matters that are critical for such advances. Many of such outside enthusiasts are former students of the Lyceum [18].

2 Signature pedagogy of engaging education

Already in 1963, in the parallel with the ideas of the teaching machine movement in the United States [19], the Lyceum started teaching informatics using its own computer. This can be seen as a quite significant achievement because not until 1985 that Soviet secondary schools started using computers in education and teaching computer programming en masse. Both authors remember that in the late 1980s, the Lyceum had its own computer laboratory where students spent much of their out of class time working on different, mostly individual, projects. It was a challenge to motivate students to stay in class, given such a strong appeal of available computers.

Nowadays, the presence in the Lyceum of unique STEM-oriented laboratories affects the curriculum of the corresponding subject matters. In particular, mathematics became much more experimental than it was before the digital revolution and this novel aspect of inquiry into mathematics affects the methods of teaching as well. One can observe that mathematical knowledge just as knowledge in science, including physics, chemistry, and biology, stems from computational experiments, and, in this regard, the words by Arnold ([20], p. 3), "despite the opinion of the majority of contemporary mathematicians, following Poincare, I consider mathematics to be part of physics, in other words, an experimental science" give special meaning to the concept of STEM. Furthermore, the notion of experimental mathematics [21,22] can be extended from the domain of mathematical research to the domain of didactics for students gifted in the subject matter so that an experiment becomes the pivot of all STEM disciplines. Although advocates of STEM curriculum integration (e.g., Moore et al. (2014) [23], Kelley & Knowles (2016) [24] and Zhu et al. (2023) [25]) do not reference Arnold's [20] view of mathematics as part of physics or Hilbert's [6] tenet that the genesis of mathematical problems is in the phenomena of real life, nonetheless the pedagogical emphasis on the value of this integration is due to the fact that dealing with real-life problems is conducive to **engaging** students in purposeful and committed learning.

This didactical perspective on teaching STEM disciplines put strong emphasis on teaching mathematics by paying attention to concrete problems that motivate topics to be taught in the Lyceum. One of the pedagogical principles in teaching integrated STEM assumes that students appreciate learning as **engaging** intellectual pursuit if they are confident in the usefulness of topics to be studied in terms of their applicability. Applicability points at the importance of concrete problems as a signature pedagogy emphasizing its deep structure when students are taught "to impart a certain body of knowledge and know-how" ([26], p. 55). Therefore, using this deep structure pedagogy for gifted students, mathematics teachers motivate the development of mathematical ideas through concrete investigations connected to other STEM subjects. In the words of Pólya ([27], pp. 100-101), a student must learn how "to recognize a mathematical concept in, or to extract it from, a given concrete situation". Following this advice, students in the Lyceum are provided with an enjoyable learning environment of mathematics curriculum comprised of carefully chosen topics to be studied and concrete problems that motivate and enhance learning of those topics. When working on such problems using mathematics, students are advised about the importance of conclusions to be aligned with geometrical, physical, biological, or engineering meaning of the problems involved. Integrated STEM may be seen as an epistemic milieu that **enriches** the described signature pedagogy.

3 Divergent production and the use of technology

Often, students can be observed demonstrating "independent creative mastery of mathematics under the conditions of school instruction" ([28], p. 68). Such conditions include allowing and even **encouraging** students to work on extensions of the traditional curriculum by posing and solving problems using their constant leaning to and interest in divergent production [10]. Whereas divergent production or divergent thinking [29] is considered as one of the aspects of creativity [30], students in the Lyceum are not offered specifically designed divergent thinking tasks; rather, they display such thinking by offering different solutions to a single task with or without any encouragement from their teachers. For example, in the case of trigonometric equations, divergent thinking may lead to posing new problems to be solved in order to navigate through the symbolic diversity of answers obtained through different problem-solving strategies [31]. One such problem is to prove formally, without using a calculator, that

$$\frac{1}{2}\cos^{-1}\frac{3-2\sqrt{3}}{3}=\tan^{-1}\sqrt{\frac{1+\sqrt{3}}{2}} \tag{1}$$

where the left- and the right-hand sides of equality (1) are different symbolic representations of a solution to the trigonometric equation

$$r+\cos^2 2x=(r-\sin x)^2 \tag{2}$$

with parameter r when $r = 2$. Equation (2) in the case $r = 2$ was included in the mathematics curriculum of the Lyceum in the late 1980s to allow for developing connections among different arc functions through a classroom communication based on discursive reasoning which, in the case of trigonometry is often counterintuitive.

In the context of American standards-based mathematics education, using appropriately designed interplay of such process standards as reasoning, communication, connection, and representation [32] in a mathematics classroom **enables** one to acquire knowledge through applying it [33]. For example, nowadays, students can use software capable of symbolic computations to see if equality (1) between two arc functions can be generalized; for example, whether the number 3, appearing in both sides of (1), may be a parameter. As shown in Figure 1 (a screen shot from *Wolfram Alpha*), the equation $\frac{1}{2}\cos^{-1}\frac{x-2\sqrt{x}}{x}=\tan^{-1}\sqrt{\frac{1+\sqrt{x}}{2}}$ has a single solution $x = 3$, implying the uniqueness of equality (1). At the same time, as shown in Figure 2, because parametric equation (2), due to the substitution $z=\sin^2 x$, turns into

$$3z^2+2\left(r-2\right)z+1+r-r^2=0 \tag{3}$$

equation (3) can be represented in a graphic form on the plane (z, r) so that when $r = 2$ we have, as shown in Figure 2 (a screen shot from the Graphing Calculator 4.1(5) where $x=z,\ y=r$), $z=\frac{1}{\sqrt{3}}\cong 0.5773502691896$, that is, $\sin^2 x=\frac{1}{\sqrt{3}}$ whence $x=\sin^{-1}\frac{1}{\sqrt[4]{3}}$, leading to another equality, $\sin^{-1}\frac{1}{\sqrt[4]{3}}=\tan^{-1}\sqrt{\frac{1+\sqrt{3}}{2}}$, to be proved with (Figure 3) or without using technology. In fact, the relevance of the latter approach to proving equalities involving irrational numbers can be seen in the expression $\sqrt{83-18\sqrt{2}}+\sqrt{2}$ offered recently by the Lyceum at the entrance exam to grade 9 with the request to simplify. This and other problems from the entrance exams (see 239.ru/vstup; accessed on August 8, 2024) indicate the Lyceum's value of paper-and-pencil mathematical simplifications.

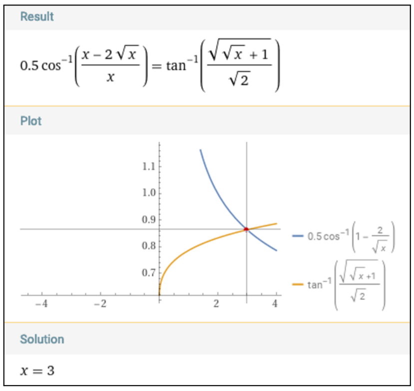

Figure 1 Using *Wolfram Alpha* for symbolic computations

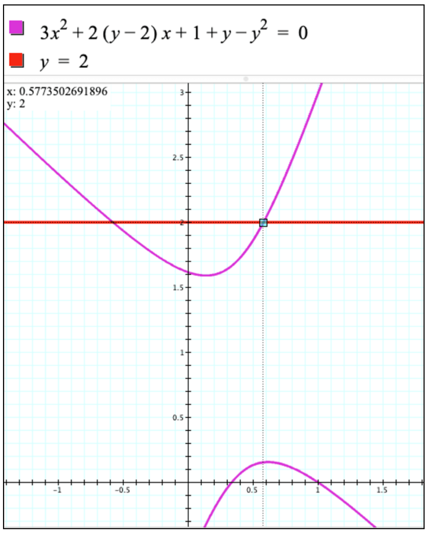

Figure 2 The graphs of the equations $3z^2+2\left(r-2\right)z+1+r-r^2=0,\ r=2.$

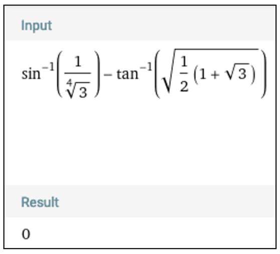

Figure 3 Proving another relation between arc functions

4 On the motivation to participate in the STEM studies

The concept of motivation represents an important factor in the description of students' participation in STEM subjects [34]. Considering two types of motivation – intrinsic and extrinsic – Biggs ([35], p. 62) admitted that intrinsic motivation in the study of mathematics is associated with "the intellectual pleasure of problem solving independently of any rewards that might be involved … [suggesting that] the aims of deep learning and of achievement motivation ultimately diverge". A classic example in support of this suggestion is a solution of the (century old) Poincare conjecture by Grigory Perelman, a former student of the Lyceum, who, after almost a decade of "deep learning", declined several international awards for his work including The Fields Medal and ($1 million) Clay Millennium Prize (https://www.claymath.org/millennium/poincare-conjecture/; accessed on August 8, 2024). This, to a certain extent, reflects the influence of cultural and national practices on the development of curiosity and independent thinking through the "deep learning" of mathematics and other subject matters [1, 36].

One student wrote about her interest in chemistry as follows: "*As a student in the school, I spend four days a week in our Chemistry Center. This place is different from Mathematics and Physics centers as here everybody knows each other, and I like the Center because by uniting students it creates a unique and very special learning milieu.*"

Another student wrote about his attraction to physics. As the student put it, "*Phenomena of science have always been appealing to me, especially those associated with physics. Physics is one of the most complicated subject matters, the mastery of which varies from individual to individual. Often, the complexity of the problem to solve and efforts spent to resolve it might force one to think about attractiveness of the problem and the desire to give up.*" But even difficult things may become likable if one can work hard.

Most of the research on the development of curiosity deal with the primary education. However, this research can inform our understanding of how curiosity turns into a motivation to **empower** a high-quality STEM professional. For example, Vidler [37] distinguished between epistemic and perceptual curiosity which are manifested, respectively, by "enquiry about knowledge … when a child puzzles over some science problem he has come across … [and] increased attention given to objects in the child's immediate environment as, for example, when a child stares longer at an asymmetrical rather than a symmetrical figure on a screen" (p. 18). Many of such children may eventually be attracted by the Lyceum and, after passing entrance exams (see 239.ru/vstup; accessed on August 8, 2024), become enrolled there at the age of eleven to study the ideas of symmetry and asymmetry across STEM disciplines. Likewise, secondary students at the Lyceum can become motivated by their mathematics or science teachers' call for questions concerning information that was shared or by their experiences with the world around them as they try to interpret "the fabric of the world … [using] some reason of maximum and minimum" (Euler, cited in Pólya (1954) [27], p. 121). In many respects, motivation to participate in the studies at the Lyceum at the level of **engagement** can be described in the tertiary level terms as "a pattern of … actions … connected with striving to achieve some internalized standard of excellence" ([38], p. 67). Just as Grigory Perelman, there are many students at the Lyceum who "are interested in excellence for its own sake rather than for the rewards it brings" ([38], p. 69) thereby demonstrating true **engagement** in the study of STEM disciplines. At the same time, many universities of St Petersburg enroll students from the Lyceum based on their excellence demonstrated during Lyceum studies and through the achievements when participating in extracurricular activities [17]. In fact, more than 50% of students in the Lyceum become accepted to universities without exams due to high quality results of their participation in Olympiads thereby demonstrating enhanced competence and sufficient knowledge level in STEM-related disciplines.

5 Enabling talents

Students in the Lyceum solve many engineering problems at the **encouragement** of their teachers. In doing so, the students often enrich existing industrial applications thereby advancing STEM research in the context of education. For example, the problem of the distant recharging of drones from power lines, developing robots for eliminating icicles, and creating autonomous system of trash collection in the ocean were solved in the Lyceum. All engineering problems require application of mathematics. Consider the problem of the construction of an autonomous soccer robot – in order to have full field observations, one must, with the help of mirror, to converge it into a single point where the video camera is located. This problem requires quite difficult computations aimed at finding the form of the surface that, due a boundary effect, is not exactly parabolic. One of the students successfully solved this problem.

Figure 4 Robots at the warehouse

Another project, dealing with robotics and developed by the graduating student (grade 11) under the supervision of two teachers of the Lyceum, deserves to be described in more detail. The project is aimed at solving several problems of contemporary robotics research – navigation of robots, designed to move objects, along the color markings on a surface with small obstructions. To solve this problem, the student developed a robot which is conceptually like an industrial robot but smaller in size. Such robots are used as equipment in a warehouse (Figure 4). Beginning from 1996, there is an international competition RoboCup – a project-oriented educational robotics milieu for high school students with the goal to increase their comfort level and mastery with technology (https://junior.robocup.org; accessed on August 8, 2024). The student created a robot for the debugging of algorithms of navigation guided by color marking and capable of recognizing and avoiding obstructions while transporting objects. The algorithms were tested at the educational competition camp RoboCup. Mathematically, the algorithms were based on some uncomplicated formulas of plane analytic geometry and used programming language C^{++} and Micro Python environment Arduino IDE. For this project, the student earned a Gold Medal at the 2023 All-Russia Robotics Olympiad.

6 Empowering teachers

As was mentioned in the introduction, one of the main aspects of the Lyceum administration governance is the creation of versatile instructional conditions that enable teachers' realization of their self-efficacy from successful learning outcomes of their students. The study in Zhu et al. (2023) [25] suggested that the integration of mathematics and science into classroom activities requires sufficient teaching time to be given to teachers. It other words, the principle of administrative flexibility [7] is what allows for an efficient teaching of STEM disciplines. This pedagogical concept is in place for a long time in the Lyceum, where all STEM-related subject matters, beginning from grade nine, are taught for two consecutive 45-minute periods. This schedule of the secondary education classes is like in the tertiary education allowing teachers to have options to combine teaching with practice and assessment. It **empowers** teachers by giving them sufficient time to do all the activities in the classroom and to answer questions by curious students. In the Lyceum, students tend to ask questions that seek explanation. Such questions are not always easy to answer. Nonetheless, teachers with many years of experience in teaching mathematics and science do know at what point a good question might be asked. Often, such teachers **encourage** students to ask questions by intentionally touching conceptually complex ideas. This strategy **empowers** both students and teachers leading to the discussion of hidden curriculum of STEM disciplines [39].

In general, teachers are **empowered** by students asking questions for at least two reasons. First, especially in the context of STEM, questions are the major means of learning, both for students and teachers. Constructivists (e.g., Steffe [40]) see mathematics teaching as a process in which both students and teachers learn. This view implies the emergence of a reciprocal learning relationship in which it is possible for teachers to learn from students as well. That is, in the question-answer intellectual milieu the two groups may learn in reciprocity. Second, by asking a question, a student may open a window to exploring new ideas and, thereby, unintentionally poses a problem [41]. The recognition of such unwitting problem posing in a question asked by a student and continuous practice in addressing problem-solving challenges **empower** teachers professionally. Indeed, the first author of this paper considers three years of teaching at the Lyceum as a truly **empowering** experience that paid off tremendously afterwards.

Having this **empowering** perspective in mind, the Lyceum is used as a practicum place for pre-service and in-service teachers of STEM disciplines. Many teachers of the Lyceum accept responsibilities of sponsors of teacher candidates enrolled in different programs at the Herzen State Pedagogical University, one of the oldest (founded in 1797) teacher education institutions in Russia. It may include not only opportunities for observation but teaching a series of lessons. Interaction with students of the Lyceum is challenging and only best of the best are sent by the Herzen University to do practicum there. Furthermore, in addition to providing in-service training for STEM teachers from other schools in St Petersburg and from different parts of the country on the regular basis, the Lyceum recently hosted an all-Russia seminar for practicing teachers of mathematics at which several teachers of the Lyceum presented their papers published in Breslav (2023) [42]. All these large-scale administrative initiatives create conditions that support successful dissemination of STEM education expertise developed at the Lyceum into classroom realities and professional development of teachers within St Petersburg (the fourth most populous city in Europe) and beyond.

7 Encouraging younger students

As it was mentioned above, the Lyceum enrolls students at the age of eleven. Young students are supervised and sponsored by older students in a number of aspects of the Lyceum studies:

providing help during young students' preparation for competitions at their level, listening and making comments when young students present their solutions to problems at mathematics and science circles [17], substituting absent teachers, sharing the rich history of the Lyceum that can be described through two veins – through the history of the school established in 1918 and through the history of Annenshule school (the building of which the Lyceum currently occupies) established in 1736 for children of German families, residents of St Petersburg. On their part, younger students were always attracted by their older peers, being impressed by their successes in different competitions at the city, national, and international levels. The presence of students in a relatively large age range requires from the administration special attention to curriculum development, after class activities, organizing summer travel and professional improvement of classes.

Summer schools of mathematics, physics, chemistry, biology, and robotics take place in camps, sponsored by major corporations including Gazprom Neft, Uralchem, and BIOCAD, represent auxiliary educational resources that are conducive to nurturing giftedness, creativity, and talent in STEM education. Whereas not all students participate in such summer activities, those who participate constitute the important body that implement knowledge gained outside of the Lyceum into classroom realities.

8 Enriching the asset of traditional skills through entrance examinations

Whereas more than 50% of students from the Lyceum earn the honor of being accepted to universities without exams, everybody, regardless of grade level, must pass Lyceum's entrance examinations. There are exams in mathematics, physics, chemistry, and biology. Formulating problems for the exams is the responsibility of the teachers of the Lyceum. The problems play the dual role: they serve as the gates into the Lyceum and used in curricula of the corresponding STEM subject matters. Despite the ubiquity of calculators and computers capable of numeric and symbolic simplifications and graphic constructions, mathematical problems for the third decade of the 21^{st} century are very similar to those offered more than thirty-five years ago when the authors of this paper were at the opposite sides of the examination table. This indicates that the Lyceum pays attention to the possession of traditional skills like paper-and-pencil algebraic simplification and plotting graphs of functions by the enrollees. Pragmatically speaking, such emphasis on epistemic value of traditional mathematical skills in the digital era makes it possible to avoid subtle errors that technology might provide. For example, some graphing utilities do not mark the points with the coordinates $x = 2$ and $x = 2/3$ as not belonging to the graph of the function which $y = \frac{3x^2-8x+4}{|2x-2|-x}$ is included in a graphing task of the entrance exam to grade 9 (the beginning of high school). Graphing functions belongs to a delicate skillset of the learners of mathematics, requiring both conceptual understanding and technological competence. Figure 5 shows the use of three digital tools – *Wolfram Alpha* (https://www.wolframalpha.com/; accessed on August 8, 2024), The *Graphing Calculator* [43] and *Maple* [44]. Only *Maple* shows the accurate graph, with a hole on each branch. This technological inconsistency among the tools suggests at least two things: the value of conceptual understanding of mathematics behind the construction of graphs that enables some initial understanding of how the final result should look like [45], and the asset of the idea of computational triangulation [46] when using more than one digital tool enhances the credibility of graphing and, more generally, of mathematical problem solving [47].

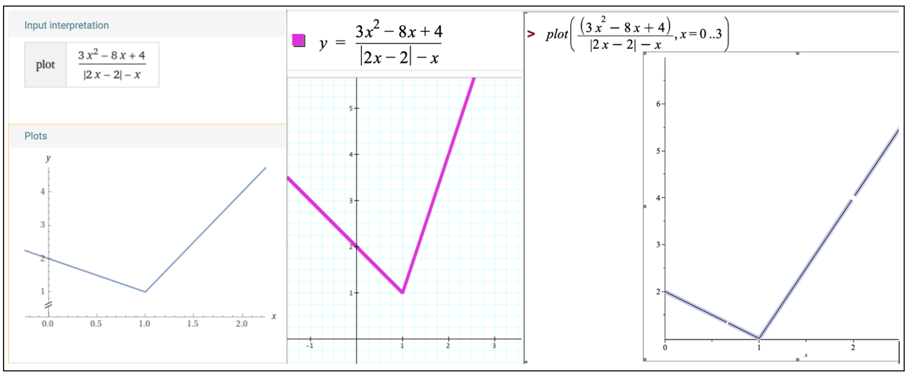

Figure 5 From left to right: *Wolfram Alpha*, *The Graphing Calculator*, *Maple*.

9 Conclusion

The paper was written to contribute to the literature about the role of giftedness and creativity in advancing the integration of STEM subjects at the middle and secondary school levels (ages 11-18). This advancement stemmed from historical, cultural, and national practices of nurturing giftedness and creativity in St Petersburg, the birthplace (1934) of the Olympiad movement in the former Soviet Union. Through fostering such practices over the years enabled the primary interest of using talent and creativity by students towards achieving excellence for its own sake who used to recognize possible rewards that excellence can bring as the secondary issue. The 5 E's theoretical framework was selected to demonstrate methods developed in the Lyceum over the span of 65 years **enabling** talents and creativity, **engaging** learning, **encouraging** divergent production of integrated subject matter, **empowering** teachers' self-efficacy through success of their students and administrative flexibility in scheduling STEM classes, and **enriching** curriculum of STEM disciplines through a combination of epistemic and pragmatic approaches to advanced pedagogy in the digital era.

To demonstrate pedagogical advances of the Lyceum, the paper highlighted signature pedagogy of using concrete problems as motivation for the study of abstract ideas across the spectrum of STEM disciplines. In particular, the teaching of mathematics as the pivot of STEM in the Lyceum emphasizes geometrical, physical, biological, or engineering meaning of the results of problem solving. Such emphasis on concrete problems attempts to provide clarification of what STEM education might mean from the point of view of STEM integration. To this end, the paper described a few mathematically enhanced robotics projects completed by students who, in their research, took advantage of the Lyceum's educationally unique laboratories of science and engineering, as well as the Center for Digital Education. In particular, the uncanny appeal of the laboratories and the Center for all students is the testament of equity and democratic access to STEM education, and especially to mathematics, in the Lyceum. It is through applying mathematical machinery by working on concrete problems that one experiences mathematics in the making and sees the subject matter through egalitarian lens.

The issue of entrance exams in mathematics was briefly mentioned in a few places in the paper. Whereas most students in STEM programs in North America are taught to rely on digital technology when plotting graphs and verification is typically due to having multiple tools producing the same graph, the Lyceum still, as in the 1980s, values traditional skills, expecting its students to be capable of graphing functions using paper and pencil. Being creative in the digital era includes understanding that technology is a tool to think with and not a tool that does thinking for the user. The case of an entrance exam task requesting to graph a function undefined at a certain point was explored using three modern-day graphing utilities. It was shown that only one of them correctly generated the graph with a hole at that point, something that future students of the Lyceum are expected to recognize through formal reasoning. This example has important implications for the professional development of teachers learning to teach or already teaching mathematical component of STEM in the digital era.

The paper recognized the important role that major national corporations and members of the STEM scientific community of St Petersburg play in supporting educational summer camps for students from all over the country and beyond. It also showed how the Lyceum participates in empowering pre-service and in-service teachers with opportunities for practicing teaching of talented and creative students and providing advanced professional development through participation in seminars where teachers of the Lyceum present their research work in STEM education. The authors see those opportunities as precious conditions that support the diffusion of STEM expertise possessed by the teachers and others in classroom realities and professional development of the wide range of pre-college educators.

To conclude note that in 2014 the Lyceum was the first school in the Russian Federation to be given the presidential status. Currently, only one more school in the country has this status since 2021 (Lyceum "Sirius" in Sochi – the largest resort city in Russia). Being entrusted with high-level national recognition of expertise in teaching STEM disciplines at the pre-college level since 1961, the teachers and the students of the Lyceum embody great responsibility and bear moral obligation to teach and learn and to be curious and creative to the best of their abilities, sharing their skills and expertise with colleagues and peers involved in STEM teaching and learning. This was also the motivation for the authors to communicate their student-teacher-administrator perspective, based on the brief and the long-term associations with the Lyceum, on how giftedness and creativity of students and advances in educational research of their teachers play out in the context of integrative science, technology, engineering, and mathematics.

Conflicts of interest

The authors declare that they have no conflict of interest.

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