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RESEARCH ARTICLE

Educational Networks as Evidence of Students' Interactions in Mathematical Learning

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Abstract: This study explores the relationship between academic performance and social network structures among tenth-grade students in Costa Rican high schools. It aims to assess how interactions within and outside the classroom correlate with mathematics grades. Data were collected from 826 students across 35 schools, and social network analysis was conducted using various network statistics. Results indicate that classroom networks, marked by higher cohesion and density, exhibited a positive but statistically non-significant correlation with academic performance, whereas outside networks showed lower cohesion and negative correlations with grades. These findings underscore the potential benefits of fostering collaborative classroom interactions to improve learning outcomes. Future research should focus on teacher-guided group dynamics and external factors influencing outside networks to gain deeper insights into their impact on academic achievement.

Keywords: educational networks, classroom interactions, studying networks

1 Introduction

In the social sciences, one of the most profound ideas is the concept of individuals as actors embedded within intricate webs of interaction, forming complex social relations [1]. Burt (2011) [2] showed that knowledge and resources are embedded within social groups and can be accessed through bridging linkages across the network. Building on this theoretical framework, the study of interactions in educational settings provides valuable insights into the social relations within classrooms.

Bruun and Brewe (2013) [3] argue that learning is a dynamic and ongoing process of transforming participation, where both the process and the participants evolve as they engage in interactions. Moreover, classroom relations can be understood as the result of small group dynamics, where a student's network position interacts with their social cognition processes [4]. This interaction occurs as students continuously evaluate and compare their own positions relative to their peers. Consequently, grades and academic performance may serve as motivators for forming social connections in school settings [4].

Grunspan et al. (2014) [5] emphasize that fostering active and participatory learning is more effective when student networks are robust, less centralized, and structured to optimize engagement. Supporting this perspective, Dou et al. (2022) [6] demonstrated that peer-to-peer interactions play a critical role in promoting active learning, which, in turn, is associated with increased self-efficacy and interest among undergraduate students in science courses.

From a different educational perspective, Rienties and Nolan (2014) [7] highlight that students from diverse cultural backgrounds develop distinct co-national and international friendships and learning relationships over time. Consequently, if cross-cultural adaptation is a goal, the role of institutions becomes pivotal [7]. Similarly, Mamas et al. (2023) [8] demonstrate that fostering denser social networks that encourage cooperation within classrooms is an effective strategy for integrating students with Special Educational Needs and Disabilities (SEND).

Grunspan et al. (2014) [5] examine social interactions among undergraduate biology students, focusing on the impact of these relationships on learning outcomes. Martinez et al. (2003) [9] take a complementary approach, employing Social Network Analysis (SNA) alongside qualitative evaluations to explore classroom social dynamics. Their work emphasizes the importance of active and collaborative learning in the educational process and aims to inform educators on improving teaching methodologies. From an applied perspective, these findings can guide educational policymakers and practitioners in refining educational strategies.

Bruun and Brewe (2013) [3] offer another compelling application of network analysis, using it to predict students' grades in subsequent courses. Their study, conducted with students in an introductory physics course at the University of Copenhagen, investigates the influence of social interactions on academic performance. By estimating network centrality measures for each student and correlating these with grades, they reveal that prior knowledge of classmates and their academic performance influences future interactions and grades in follow-up courses. Considering the importance of understanding student interactions to enhance learning processes, this study applies Social Network Analysis (SNA) to examine how social relations among students may influence their academic performance. The study uses data from the formal Costa Rican educational system.

Studies from various regions have employed Social Network Analysis (SNA) to explore students' interactions and learning processes. For instance, Mullick et al. (2023) [10] provide evidence that identifying influential individuals through SNA is essential for understanding communication patterns and advice-seeking behaviors in schools in China. Similarly, Rakic et al. (2018) [11] demonstrate that students' interactions on e-learning platforms in Serbia are significantly associated with their academic performance. Maleko et al. (2011) [12] present the case of integrating interconnected learning communities with traditional educational systems in Africa, showing that fostering social interaction and collaboration leads to improved learning outcomes.

This research holds particular significance for Costa Rica and Latin America, where discussions about educational investment remain at the forefront. Moreover, there are challenges associated with studying learning environments and student performance in Latin America, primarily due to the difficulties in collecting data at both regional and national levels [13].

Queupil (2016) [14] examines educational collaboration networks and leadership in the region, focusing specifically on Chile. The study highlights that Costa Rica and other Central American countries are largely disconnected from the broader regional collaborative network, in contrast to Mexico, Argentina, and Brazil, which exhibit higher levels of inter-country collaboration. In Chile, regional disparities are evident, with schools near the capital demonstrating stronger collaboration compared to those in more remote areas. In this context, enhanced collaboration fosters greater information exchange, which has the potential to improve school performance and educational outcomes.

According to ECLAC (2022) [15], countries with the highest educational investment in the region include Cuba, Bolivia, and Costa Rica. Despite its small size, Costa Rica allocates approximately 6% of its GDP to education, primarily for public preschool, primary, and secondary education. However, the quality of this investment has declined over the past decade.

Costa Rican students' performance on PISA examinations has been poor compared to developed countries and other Latin American nations such as Uruguay, Mexico, and Chile. In particular, mathematics results from PISA indicate that most students score at minimum levels, demonstrating weak problem-solving and logical thinking skills. These findings align with the outcomes of national assessments administered to secondary school students [15].

Glewwe and Kremer (2005) [16] underscore the complexity of the relationship between economic investment and educational outcomes. Examples from developing countries, such as Kenya and Mexico, highlight how an emphasis on increasing the quantity of education can sometimes compromise its quality. Educational outcomes are shaped by various factors, with Buchmann and Hannum (2001) [17] identifying family, school, and community dynamics as central. The present study specifically examines the influence of social networks, which intersect with both school and community factors, providing critical insights into their role in enhancing educational achievements.

The objective of this research is to identify relational patterns among students and explore how these relationships potentially correlate with academic performance in mathematics. Following Bruun and Brewe (2013) [3], we define social interaction as moments when students exchange ideas, actively use mathematical language, and critically evaluate their classmates' arguments both inside and outside the classroom. These attributes of social relations are hypothesized to be linked to better grades and academic performance. Therefore, we anticipate a positive relationship between educational performance and denser, better-connected networks within the classroom, as well as the students' learning networks outside the classroom. This study is particularly significant because research applying SNA to educational systems in Latin America is rare. Furthermore, comparisons of networks at the classroom level are scarce due to the challenges and costs associated with data collection, as noted by Grunspan et al. (2014) [5].

2 Materials and Methods

We propose using Social Network Analysis (SNA) to highlight student relational patterns. These patterns are identified through network centrality measures across two distinct types of networks. Academic performance, on the other hand, is represented by the students' grades within each classroom, as detailed in the following sections.

2.1 Research Method

We applied Social Network Analysis (SNA) to examine interactions such as studying together or collaborating during class. This study focuses on two distinct networks: the outside classroom network, which captures co-studying relationships formed when students prepare for mathematics exams outside school, and the inside classroom network, representing co-working relationships developed through classroom interactions.

These two networks have been shown in the literature to have distinct but significant influences on student behavior. Outside the classroom, groups facilitate social interactions, while the classroom provides a controlled environment where collaborative relationships can be fostered [5]. These relationships, whether formed inside or outside the classroom, have demonstrated meaningful effects on students' behaviors and outcomes [3, 5, 6, 8].

By applying questionnaires to students, it was possible to collect names to reconstruct their ties. This approach involves students naming their classmates in response to specific questions, enabling the researcher to identify links between the respondent and the classmates mentioned. As a result, a personal network for each student is created and introduced into a database in the form of an adjacency matrix. An adjacency matrix is a square matrix where each cell represents a potential connection between two students: a value of 1 indicates the presence of a relationship, and a value of 0 indicates its absence. By combining all individual connections, the complete network for a classroom can be represented.

There are various methods to elicit ties and recall names, and the most suitable approach depends on the characteristics of the network and the type of respondents. For example, if a network is dispersed and includes links to external members, it is recommended to use open-ended questions to generate contact names [18]. This approach is particularly advantageous for collecting data from small to medium-sized networks [19].

The present analysis focuses on interactions among students within a classroom, representing a closed network that considers only ties between classmates. For such cases, it is recommended to conduct a census, meaning that all students in the classroom are surveyed to build a complete representation of the network [20]. In Costa Rica, previous studies indicate that each classroom typically constitutes a closed and small network, with the number of students ranging from 15 to 35 [21]. Based on a predefined student list, a complete map of relationships between students was constructed.

The research instrument included two questions. The first focused on identifying which classmates the student interacted with outside the classroom while studying for mathematics (answers were strictly limited to classmates' names). The second question centered on which classmates the student interacted with inside the mathematics classroom when coworking was needed. These questions were designed following recommendations from the literature to ensure clarity and avoid ambiguity. Additionally, respondents were not restricted in the number of names they could provide, as suggested by Grunspan et al. (2014) [5] and methodologically analyzed by Hennig et al. (2012) [22].

The research faced certain limitations, such as students having the same name or respondents being unsure of their classmates' surnames. To address these issues, participants were allowed to provide additional descriptions to clarify their responses. This ensured that the person administering the instrument could verify and resolve any ambiguities at the end of the questionnaire, as recommended by Grunspan et al. (2014) [5].

2.2 Field research

The field research was conducted in coordination with the Department of Education of Mathematics at the Universidad Estatal a Distancia (UNED), Costa Rica. The study focused on fourth-year secondary school students. In the Costa Rican educational system, students complete six years of elementary school followed by five years of secondary school, with secondary school serving as the final stage before university. The instrument was applied in a single classroom within each of the 32 selected educational centers.

The sample was drawn from high schools in Costa Rica's Metropolitan Area, specifically including public daytime academic high schools with an enrollment of 500 or more students. The final sample consisted of 30 public and 5 private high schools. In each school, a tenth-grade class was randomly selected. In total, the sample encompassed up to 826 students.

A key methodological constraint was that the research initially aimed to include 35 educational centers. However, a general strike by public employees occurred before the fieldwork concluded, preventing access to three schools. Consequently, the instrument was applied in 32 schools instead. The instrument was administered between February and October 2018. On average, it took about 15 to 20 minutes for students to complete the research instrument, including time for explanation and introduction. Additional questions revealed that 90% of the students chose their study and co-working partners. In the remaining classes, either the teacher or both the teacher and students made the selections.

To validate the efficacy of the social network instrument, a pilot application was conducted with a selected classroom from a secondary school that was not part of the final sample, as recommended by Grunspan et al. (2014) [5]. The pilot test evaluated the effectiveness of the questions, the time required for completion, and the clarity of the instructions. This process helped refine the instrument and improve its application.

To assess the correlation between academic performance and network structure, a mathematical knowledge test was administered to all students in the selected classrooms. The test covered content from the Ministry of Education's curriculum, including statistics, probability, algebra, and numerical and geometric relations [23]. More specifically, the test items were sourced from the national test item bank managed by the office of educational quality assessment (DGEC-MEP) of Costa Rica's Ministry of Education.

The grading scale ranged from zero to ten, with ten being the highest possible grade. Overall, performance was poor, with the best score being seven and the average score around four. For analytical purposes, grades between 4 and 6 were classified as reasonably acceptable (above average), while a score of seven was considered the highest achievement.

Finally, all research instruments underwent rigorous validation by experts in mathematics education, who are also researchers and instructors at Costa Rican public universities. The mathematical test utilized validated and pretested items provided by the DGEC-MEP. Additionally, the test was piloted with tenth-grade students to ensure the clarity and comprehensibility of the instructions, enhancing its reliability and applicability.

2.3 Data analysis

The data was analyzed using the statistical platform R, a language and environment for statistical computing and graphics [24]. Within this framework, the STATNET suite of packages [25] was used, which specializes in network analysis.

In network analysis, statistics are used to describe the position of a student within a network, commonly referred to as centrality measures [3]. Centrality in a network indicates the position an actor holds within the network, reflecting how the student is either sought after or actively seeks out classmates to study or collaborate in the classroom.

Therefore, an actor with many relations is a high central actor [26, 27]. The measure is estimated as follows: $Deg_i = \sum_j a_{ij}$, the degree centrality, Deg_i , is the degree of *i*, that is the addition of all links between *i* and other students [28].

The second centrality measure used was the betweenness centrality. The betweenness represents a statistic that denotes how central an actor is as the actor takes a position of bridge or stays between other actors. The next formula shows the betweenness centrality [26]: $C_{bi} = \sum_{i \neq s, i \neq j, j \neq s} \frac{b_{isj}}{b_{ij}}$, where, b_{isj} are all the possible shortest paths between *i* and *j* that passes through actor *s*, and b_{ij} are all the possible shortest path between *i* and *j*. In other words, the statistic indicates the students that are central at the moment to connect other students, representing a brokerage role.

A third measure used is the network density, this statistic indicates the proportion of number of ties in the network given the potential number of ties possible [29]: $d = \frac{links}{\frac{n(n-1)}{2}}$, where *d* is the network density, *links* is the total of relations in the network, and *n* is the number of students that participate on the network. Essentially, this measure counts for the realization of ties given the potential that the network has if all students were studying with each other. However, in the network there may be "isolate" students who prefer to study alone or work alone in the classroom, this influences the density downwards.

Figure 1 illustrates the network statistics described earlier. For demonstration purposes, the case of three nodes is presented for network cohesiveness and density, although the same principles can be extended to networks with a larger number of nodes. A high degree centrality indicates that a student has many connections with their peers. High betweenness centrality represents a student who acts as a bridge, connecting multiple study groups. High density, on the other hand, suggests that the network has a large number of possible ties, which in turn indicates a more cohesive structure compared to a network with lower density.

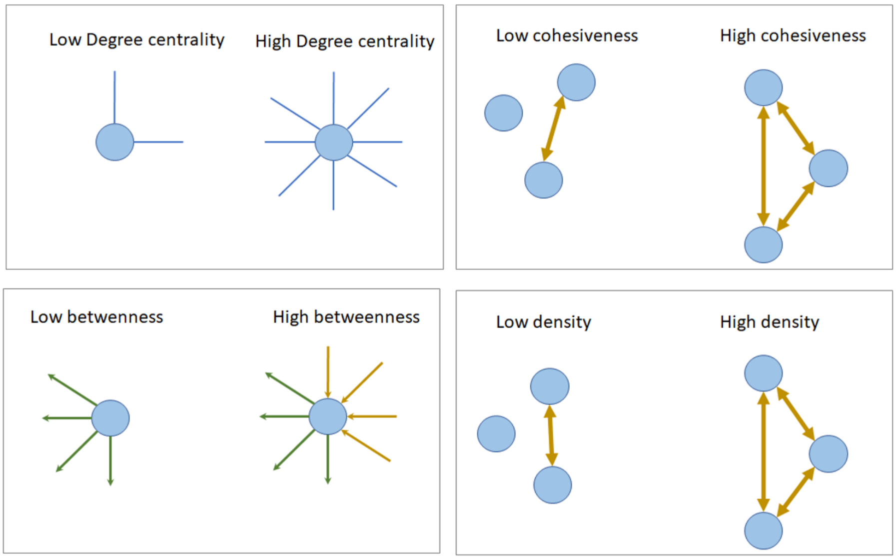

Figure 1 Social Network Analysis Concepts

For each of the two networks, centrality measures were estimated for each student and then correlated with their grades on the mathematics examination. Additionally, an Analysis of Variance (ANOVA) was conducted to assess differences between classrooms within the same network type.

2.4 Ethics

The questionnaire was administered with prior approval from the Ministry of Education, in collaboration with the educational centers and their directors, and with the consent of both the students and their parents. To ensure confidentiality, respondents were informed about how the collected data would be used. Additionally, it was explained that all data would be completely anonymized, with each student's name replaced by an ID number.

3 Results

3.1 Describing the Learning Networks

Figure 2 illustrates the network representations of both the inside-classroom network and the outside-study network. These networks are composed of the total student relations from the 32 schools, with each color representing a different classroom. Overall, the inside-classroom network appears more cohesive and contains denser groups compared to the outside-study network.

However, when examining the network structure of individual classrooms, different patterns emerge. For instance, in the school with Id33, the outside network exhibits a higher density, while the inside network has fewer ties and many isolated nodes. This suggests that the dynamics of working together within the classroom are not as dense as those of the outside network. In this case, most students seem to work individually within the classroom (see Figure 5 for details).

Figure 3 illustrates the concentration of central network positions among students, as mea-

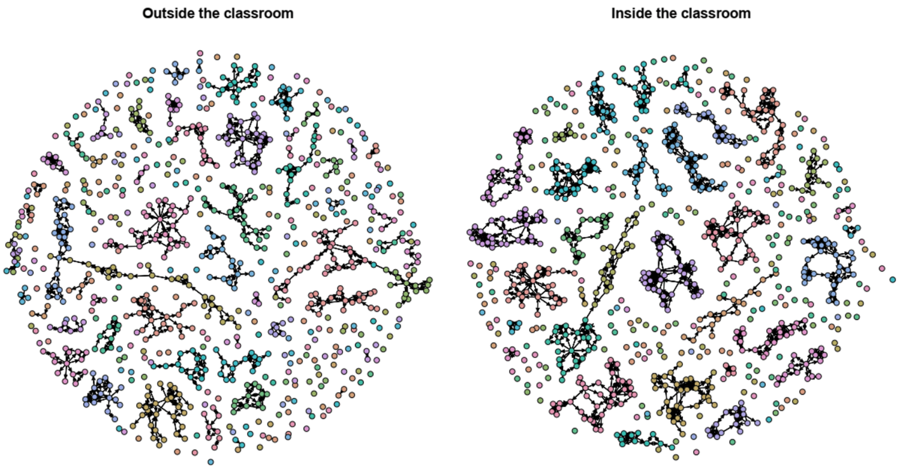

Figure 2 Learning Network inside and outside the classroom

sured by betweenness centrality. In the outside network, students who act as bridges between studying groups are highly concentrated. This concentration is represented in the Lorenz Curve, which shows the distribution of network centrality positions. For example, 10% of the students account for 85% of the bridging or brokerage value in the outside network. This indicates that only a small number of students interact with multiple study groups, serving as bridges between them. This is significant because the exchange of information flows through these bridges, giving these students a comparative advantage. They have access to diverse information from different groups, which is not available to those who interact with fewer groups.

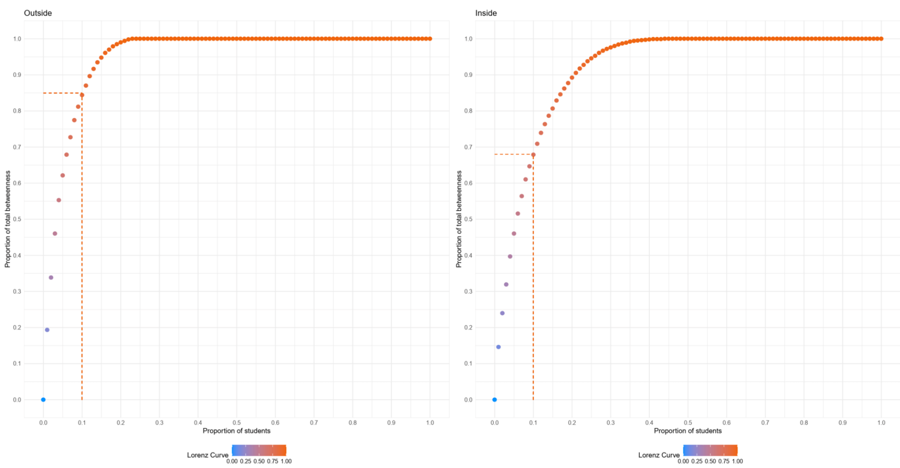

Figure 3 Concentration of bridging position (centrality betweenness) in the network: outside network on the left side and inside network on the right side

In the case of the inside network, 10% of the students account for 68% of the bridging value. This suggests that the bridge function is distributed among more students compared to the outside network. It indicates that in the classroom, a single student may belong to multiple co-working groups, which is more common than in the outside network. As a result, this helps to democratize knowledge, as one student is likely to share their mathematical knowledge and information across various classroom groups. This facilitates a broader exchange of knowledge among multiple working groups.

Figure 4 shows the distribution of nominations among students in both the outside and inside networks. In the outside network, 30% of the students account for 75% of the links, while in the inside network, 30% of the students hold 63% of the ties. The concentration of links is more evenly distributed in the inside network, indicating that, as the proportion of links increases, the concentration of nominations shifts towards fewer students.

Table 1 presents key statistics that highlight the differences between the two networks. Inside the classroom, students form more ties with each other, with an average of about two more co-working connections compared to their outside study group. This is reflected in the higher network density.

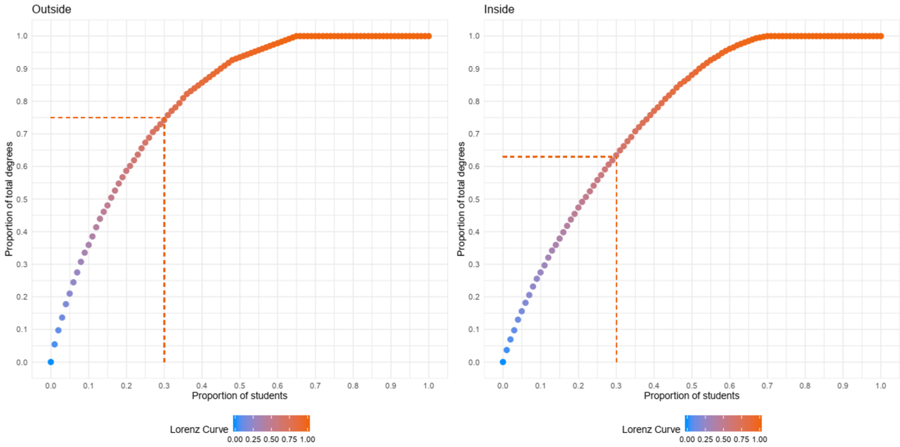

Figure 4 Concentration of centrality degrees in the network: outside network on the left side and inside network on the right side

Table 1 Networks' statistics

Statistics	Outside the classroom	Inside the classroom
Number of Students	826	826
Number of Relations	1262	1750
Density	0.18	0.26
Isolates (%)	288 (35%)	245 (27%)
Mutual relations (%)	209 (16.6%)	484 (27.7%)
Mean Ties per student	2.29	4.16

Source: self-elaboration based on collected data.

Moreover, there are students who prefer to study and work alone. The proportion of isolated students in the outside network is approximately 35%, while it is 27% in the inside network, meaning there is an 8% higher proportion of isolated students in the outside network. This can be seen in Figure 4, where the curve flattens once it reaches 1 on the y-axis, indicating the point at which ties stop growing and isolated students emerge. Additionally, the results show that mutual ties account for about 16.6% of the total ties in the outside network, compared to 27.7% in the inside network.

3.2 Classrooms' Network Differences

To estimate differences between schools, the networks for each classroom were analyzed separately for both the outside and inside networks, as shown in Figure 5. Observable differences emerge between the two networks within the same classroom. For instance, the school with id31 exhibits a well-connected, cohesive, denser, and more structured inside network compared to its outside network. This indicates that students in classroom Id31 collaborate effectively within the classroom, with at least three students acting as brokers, connecting different subgroups and bridging these co-working groups. In contrast, the outside network for the same classroom presents a starkly different structure, with four disconnected study subgroups and five students working alone.

Moreover, when comparing the inside network classrooms, an ANOVA test for betweenness centrality reveals statistically significant differences (p-value = 6.83e-10), with a post-hoc Tukey multiple comparisons test (95% confidence interval) identifying pairwise differences. For example, significant differences were found between classrooms id05 and id01, id10 and id01, and id32 and id01, among others. Similarly, the degree centrality distribution for classrooms in the inside network also shows statistical differences according to the ANOVA (p-value $<$ 2e-16), with a post-hoc Tukey test identifying pairwise differences. Notable differences were observed between classrooms id08 and id05, id25 and id05, id15 and id06, among others.

Similar results were found in the outside network. Nonetheless, in the outside network, there are more statistically significant differences in the network measures when comparing the ANOVA post-hoc test. In general, the betweenness centrality post-hoc test for the inside network identified 35 pairwise differences, and for the degree centrality in the inside network, 58. For the outside network, 77 pairwise differences were identified for the betweenness centrality and 231 for the degree centrality. This indicates that the network configurations of the classrooms have more differences in the outside network than in the inside network. These network configurations are shown in Figure 5.

There are cases where the outside network and the inside network are quite similar. In these cases, the interaction between students inside and outside the classroom is essentially the same, from a network perspective. Two clear examples are id11 and id05.

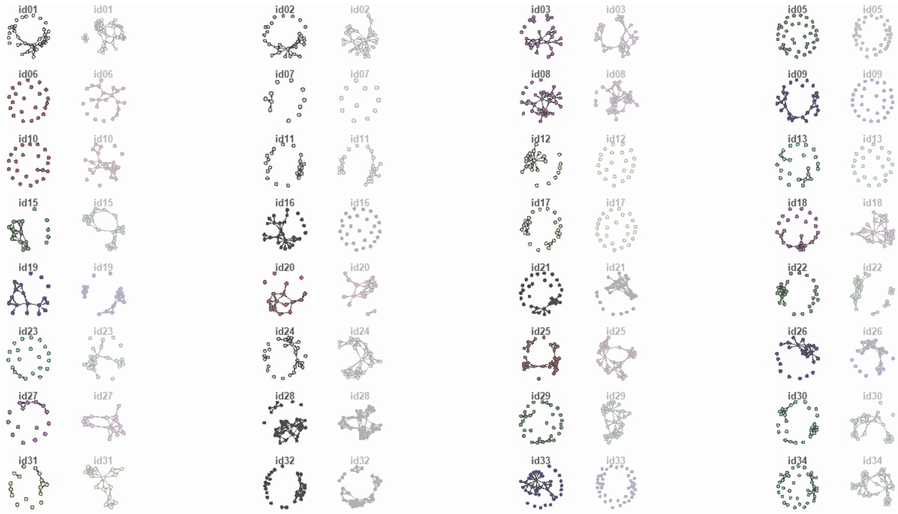

Figure 5 Learning Networks: *outside* versus *inside* the classroom (*inside* networks in water mark)

Figure 6 shows the positions of each school when comparing the inside to the outside network densities. Based on this comparison, we can categorize the classrooms into four types, as follows:

(1) First case: Left-upper quadrant – Classrooms that create denser co-work networks inside the classroom than outside. There are 13 classrooms in this category, representing 41% of the total.

(2) Second case: Left-lower quadrant – Classrooms where both network densities are low. There are 16 classrooms in this category, representing 50% of the total. These classrooms exhibit low interaction both inside the classroom for co-working and outside the classroom for study. Special attention should be given to seven cases with zero co-work interaction inside the classroom. This is the least desirable scenario from an interactive co-learning perspective.

(3) Third case: Right-lower quadrant – Only one classroom (3% of the total) falls into this category, where the outside study network is denser than the inside co-work network, and inside interactions are absent. Classroom id07 is in this category, where the outside network is more connected, while inside interactions are non-existent.

(4) Fourth case: Right-upper quadrant – The most desirable scenario, where both the inside and outside networks are highly dense. This indicates a rich exchange of knowledge and the diffusion of experiences across two different learning environments. However, only two classrooms (6% of the total) are in this category: id15 and id20.

To define the correlation between the network densities and the students' grades, we present the results graphically in Figure 7. The findings reveal a negative correlation between the outside network statistics and the grades. In contrast, there is a positive correlation between inside network density and grades (p-value = 0.1100, Pearson Correlation), although it is statistically not significant. This can be observed in Figure 7, where classrooms id24 and id08, which have the best average grades, also exhibit higher inside network densities, especially classroom id24. As Figure 7 illustrates, most classrooms are clustered around a grade of 5, which represents the middle point. The average grade was 4, so grades between 4 and 6 are considered acceptable.

When examining the correlation between the average degree centrality of the classrooms, we observe a positive correlation (0.4671, Pearson Correlation) between the inside and outside network centralities, which is statistically significant (p-value = 0.0070). A similar relationship is found between the betweenness centralities of the inside and outside networks (0.4664, Pearson Correlation, p-value = 0.0072). This indicates that higher degree and betweenness centralities in the inside network are associated with higher values of these statistics in the outside network.

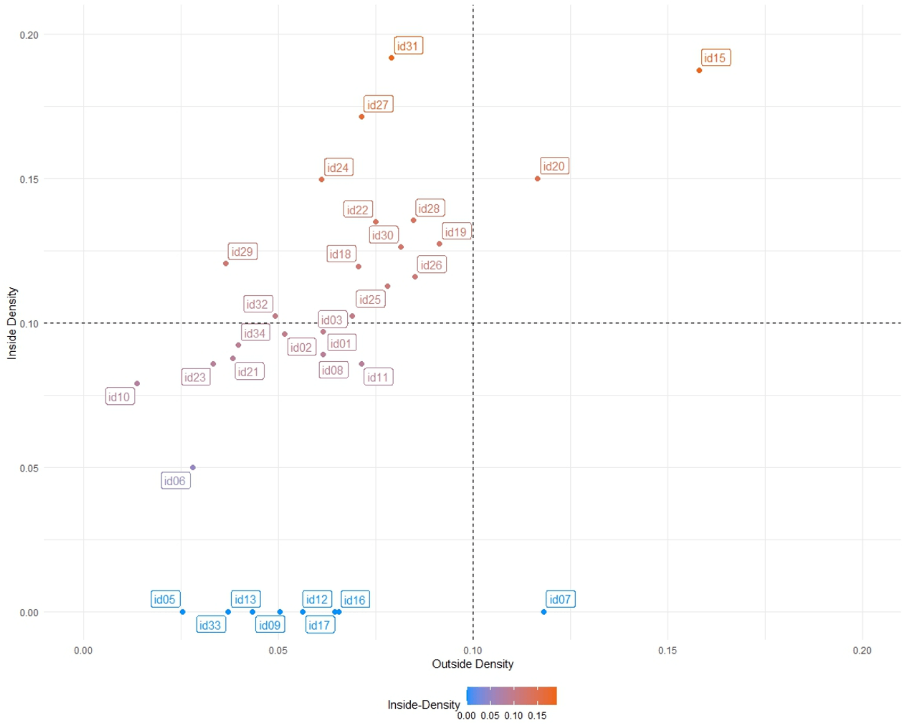

Figure 6 Comparing inside and outside network densities

Other factors, such as the quality of interactions, the quality of study materials, and the teacher's ability to promote effective knowledge transfer, are not directly observable in the data. However, the Pearson correlation test shows a positive correlation between inside network densities and classroom average grades (0.031, Pearson Correlation), although the correlation is low and not statistically significant (p-value = 0.86). In contrast, the outside network densities are negatively correlated with the classroom average grade (-0.167), but this is also not statistically significant (p-value = 0.3589).

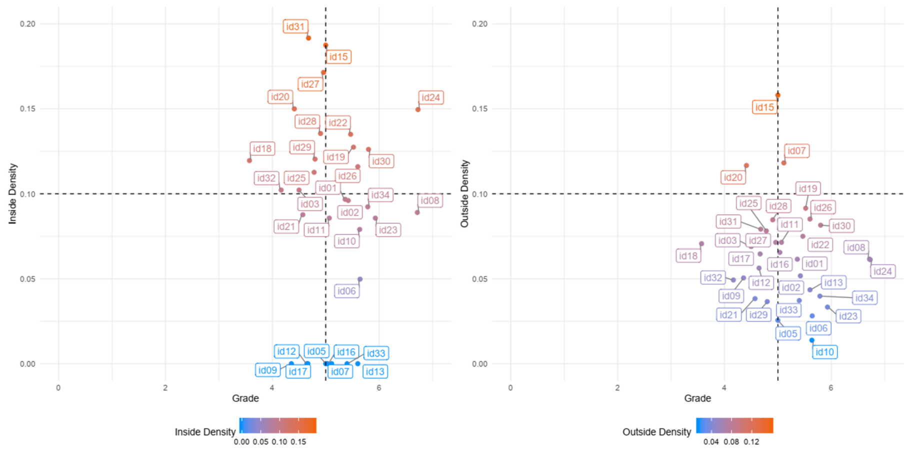

Figure 7 Densities inside and outside against the average grade of the classroom

The correlation of degree centralities for the inside network exhibits a similar trend to network density. The degree centrality correlation with grades is 0.115 (Pearson Correlation), but it is not statistically significant (p-value = 0.5315). In the outside network, degree centrality is negatively correlated with grades (-0.08, Pearson Correlation), and this correlation is also not statistically significant (p-value = 0.664).

Regarding betweenness centrality, the inside network shows a positive correlation with grades (0.1127, Pearson Correlation), but again, it is not statistically significant (p-value = 0.5392). Conversely, the betweenness centrality in the outside network is negatively correlated with grades (-0.0936, Pearson Correlation), but this relationship lacks statistical significance (p-value = 0.6103).

4 Discussion and Conclusion

4.1 Discussion

Several authors have demonstrated that positive learning and coworking relationships enhance academic performance. Thus, analyzing the relationship between grades and network configurations is highly relevant. As highlighted by Rienties et al. (2014) [7], higher density and reciprocity within a study network indicate psychosocial support, a sense of belonging, and reinforcement of identity and recognition, all of which enhance the learning process. Since students are embedded in social groups through network relationships, their learning network significantly influences their attitudes and behaviors [4]. This is particularly important because students' interactions also involve social cognition processes [4]. Continuous contact with peers is associated with the development of a student's social interpretation of their own position within a group [4]. Consequently, Rienties et al. (2014) [7] emphasize that fostering study relationships among students should be a key goal to improve the learning process and academic performance.

Our results reveal a positive correlation between grades and active participation within the inside-studying-network, as demonstrated by the revised statistical analysis. However, it shows a negative correlation between the outside-studying-network. This may indicate that the context and environment where the interaction takes place may influence the outcome. Classroom coworking is positively associated with grade outcomes, with success relying on fostering a supportive atmosphere of social interaction around the learning process in mathematics. In this context, Rienties et al. (2014) [7] demonstrate that, for international students, difficulties in establishing robust coworking and study networks represent a significant barrier to integration.

Additionally, Dou et al. (2022) [6] demonstrate that students with high network centrality exhibit stronger self-efficacy in physics learning. An active learning environment establishes conditions that positively engage students, where their social position within the classroom network fosters confidence, particularly in the context of physics education. Enhanced teacher involvement can further promote the creation of these social ties, thereby strengthening learning processes.

Our analysis shows that classroom networks are more interconnected and cohesive than networks outside the classroom. Within the classroom, students may feel a sense of obligation to collaborate with their peers, or a more conducive learning environment is fostered, as demonstrated by Dou et al. (2022) [6] and Mamas et al. (2023) [8]. Conversely, at home, some students may prefer to study alone or interact with only a few classmates. This preference could stem from various factors, such as the distance between their homes and those of their peers, sociability, personality traits, or socio-economic circumstances.

Notably, 27% of students work alone inside the classroom, a lower percentage compared to the 35% who study alone in the outside-classroom network. The presence of isolated students in the classroom could be attributed to a combination of factors, including personality traits, sociability skills, or a lack of effective teacher intervention and organization. More specifically, the presence of isolated students reflects a lack of ties within the network, while mutual ties are indicative of reciprocity. Our results indicate that mutual ties represent 16.6% of the total ties in the outside network but increase to 27.7% in the classroom network, underscoring the greater cohesiveness of classroom interactions.

To understand these findings, it is important to note differences among schools. Some schools have smaller class sizes and actively promote cooperative work within the classroom, while others do not. Although the Ministry of Education advocates for group work in classrooms, not all teachers implement this practice consistently [19].

The ANOVA post-hoc tests for betweenness centrality revealed statistically significant differences between schools. For the inside-classroom network, 35 pairwise differences were identified, whereas 77 pairwise differences were found for the outside-classroom network. Similarly, for degree centrality, the inside-classroom ANOVA post-hoc test identified 58 pairwise differences, while the outside-classroom test revealed 231 pairwise differences.

These statistical results suggest that classroom networks exhibit greater variation in the outside-classroom context compared to the inside-classroom context. Inside-classroom networks tend to be more homogeneous, as students engage more uniformly in collaborative activities during mathematics lessons. In contrast, outside-classroom networks are more diverse, with some schools exhibiting high network density and others showing very low density.

Additional network measures further support these distinctions. For example, the mean number of ties per student in the inside-classroom network is 4.16, nearly double the mean of 2.29 ties per student in the outside-classroom network. This indicates that a stronger co-working dynamic occurs within the classroom, underscoring the notion that students are more inclined to collaborate in structured, controlled environments such as classrooms than in less regulated, outside-classroom settings.

As previously mentioned, the results indicate a positive correlation between classroom average grades and network metrics such as density, degree centrality, and betweenness centrality within the inside-classroom network. In contrast, these same metrics exhibit a negative correlation with grades in the outside-classroom network. This suggests that, within the classroom network, higher numbers of relationships and bridging ties are associated with better academic performance.

Conversely, the negative correlation in the outside-classroom network may reflect a more complex origin of relationships. Specifically, students with higher grades appear to build fewer connections in the outside-classroom network. These findings highlight a distinction in the social dynamics of studying mathematics: the individuals with whom a student interacts inside the classroom differ from those they engage with outside the classroom. In line with these observations, Dou et al. (2022) [6] assert that the nature of a student's interactions significantly influences their academic outcomes. This reinforces the importance of examining not only the quantity but also the context of peer relationships in fostering effective learning environments.

Different social processes underlie classmates' collaboration. Social retribution and recognition may influence the formation of learning ties [30]. Furthermore, homophily – where relationships form between students who share similar attributes, such as grade averages, gender, or family ties—can play a significant role in determining whether a connection is established [4]. These factors are particularly relevant given the inherent difficulty in measuring a student's effort in forming relationships, both inside and outside the classroom [5].

According to Buchmann and Hannum (2001) [17], educational inequalities among students arise from various factors influencing academic performance. Outside the school, family factors are emphasized as critical in supporting students' learning and providing essential resources. Additionally, the community's social structure and supporting networks play a significant role. Within the school environment, factors such as the conditions of educational centers, social processes, and institutional organization are pivotal. The social network within the classroom is deeply embedded in the social learning process, as also recognized by Friedkin and Johnsen (2011) [4]. Understanding the external influences on learning, including interactions outside the school, presents a challenge for future research. Such studies should also incorporate the role of kin networks and community-support networks in shaping students' educational experiences and outcomes.

The ideal scenario for fostering effective learning environments occurs when both inside- and outside-classroom studying networks are strong and dense, characterized by numerous relationships among students. Figure 6 illustrates this optimal case in the upper-right quadrant, where both networks exhibit high density. In such cases, there is a greater potential for the exchange of knowledge and the diffusion of experiences, as more ties are established and overlap across the two learning environments. However, this scenario is rare; only two classrooms (6% of the total sample, identified as classrooms id15 and id20) occupy this quadrant.

There is a significant positive correlation between network metrics for inside and outside networks. For instance, higher degree centrality in the inside network is associated with higher degree centrality in the outside network (correlation = 0.4671, p-value = 0.0070). However, notable exceptions exist. Specifically, 41% of the schools exhibit high inside-network density but low outside-network density.

The denser inside networks indicate that students are willing to collaborate and actively engage in co-working within the classroom. This contrasts with the lower interaction levels observed in outside networks, which, while less dense, still maintain a meaningful degree of connectivity. The classroom environment, characterized by physical proximity and a structured, controlled learning setting, facilitates easier interaction. Consequently, students are more likely to collaborate with a diverse range of peers during in-class activities. This dynamic fosters enhanced learning exchanges among students. In addition, the classroom offers students similar educational resources, which stimulates a less unequal environment and potentially more equitable learning outcomes [11]. This is consistent with the grade results observed in our study.

Thus, increased interactions within the classroom lead to improved academic performance.

When aligned with factors such as a teacher's positive disposition, sufficient study materials, and favorable physical conditions [16], the learning process can be significantly enhanced. This effect is further amplified when external factors, such as family and community support networks, are structured into positive, reinforcing systems that benefit students [17]. These considerations are essential for designing comprehensive educational policies aimed at fostering academic success and promoting equitable learning opportunities for all students.

Another important factor in the social dynamics within education is the students' network position. The relatively homogeneous degree centrality suggests that no single "popular" student dominates classmates' selection. However, high betweenness centrality highlights the importance of sociability in bridging different cohesive study groups and fostering classroom collaboration. This underscores the value of specific network positions in maintaining dense and cohesive study groups.

The Lorenz concentration curve for degree centrality and betweenness centrality reveals a lower concentration of highly central actors within the classroom compared to the outside studying network. This indicates that central students dominate the network more prominently in outside interactions. In contrast, classroom interactions feature more bridges (lower concentration of betweenness centrality), fostering a more homogeneous distribution of information. Mullick et al. (2023) [10] highlighted the pivotal role of highly central actors in providing advice within schools. Their findings align with ours in demonstrating that high reciprocity and balanced degree centrality effectively facilitate the exchange of knowledge and skills among students. Moreover, Leithwood (2010) [31] emphasizes the central role of school leadership and collaborative work as key strategies in transforming underperforming schools.

Students occupying these central roles can be regarded as key actors for teachers to engage when aiming to foster new ties or implement integration strategies, as also Mullick et al (2023) [10] found. In our case, these students typically combine satisfactory academic performance with sociability recognized by their peers. As Dou et al. (2022) [6] found, the number of classroom interactions and the characteristics of the students involved in these interactions significantly impact self-efficacy development in physics learning. Additionally, Mamas et al. (2023) [8] emphasizes that fostering assertive learning interactions within the classroom can help integrate less advantaged and isolated students, promoting a more inclusive learning environment. These results have significant implications for teachers and education policymakers, highlighting the importance of fostering positive leadership within the classroom and promoting its extension to interactions beyond the classroom environment to enhance overall learning outcomes.

The correlation between inside-network density, degree centrality, betweenness centrality, and students' grades was positive but not statistically significant. Since these correlations were estimated at the classroom level for comparative purposes, the heterogeneity among classrooms may have influenced the results.

Nonetheless, this finding tendency aligns with Bruun and Brewe (2013) [3], who demonstrated that students' positions within communication and interaction networks correlate with academic performance, particularly among university physics students. Bruun and Brewe reported highly significant correlations ($p < 0.001$) between network centrality measures and grades across all networks. Their results emphasize the interplay between social and academic interactions in the classroom, which they consider an integral part of the learning process.

Similarly, Grunspan et al. (2014) [5] highlighted the existence of socially disconnected students who may prefer studying alone at home, believing it to be a better strategy. However, within the classroom environment, each link in the study network potentially represents the bidirectional exchange of class material. In this context, socially engaged students may have an advantage over those who do not actively share or discuss class material with peers. Our results align with these findings, as the inside network has a lower percentage of isolated students (27%) compared to the outside network (35%), indicating a higher level of collaboration and study interaction within the classroom network.

Akin to the findings of Bruun and Brewe (2013) [3], our study indicates that students who occupy more central positions within the classroom problem-solving network, sharing their knowledge, also tend to achieve higher grades. Although our study did not observe a large number of students with high grades, those with above-average performance (grades between 4 and 6) were more frequently sought out by their classmates for co-learning in the inside network, which demonstrated higher network densities. This aligns with Bruun and Brewe's (2013) [3] assertion that academic meaning-making occurs as students recognize classmates who actively engage in collaborative practices.

Supporting this perspective, Martínez et al. (2003) [9] revealed that in-classroom collaboration, co-working, and information sharing are influenced, at least in part, by the relationships formed around shared academic goals. However, these authors caution that classroom dynamics are more complex than they may appear. Their study found that only a minority of students (16.5%) considered their classroom environment to be collaborative, while 30.6% reported that collaboration occurred primarily with friends. Despite these findings, most students expressed a willingness and motivation to work in groups. This contrast suggests a discrepancy between students' appreciation of collaboration as an abstract value and their limited practical experience, which does not always foster collaborative attitudes.

Because of this, Martínez et al. (2003) [9] highlight the teacher's central role as a catalyst for interaction in the in-class learning process. Teachers play a vital role in organizing and fostering positive group co-working and collaborative experiences in the classroom. This role involves facilitating knowledge exchange, encouraging positive social cognition processes, and promoting academic meaning-making through cohesive social networks. Mamas et al. (2023) [8] further emphasize the importance of the teacher's role in enhancing the frequency of interactions involving students with Special Educational Needs and Disabilities (SEND). Teachers, through intentional interventions, can stimulate cooperation among students, thereby reducing isolation and exclusion.

On a broader scale, Penuel et al. (2006) [32] argue that schools where teachers collaborate more frequently tend to achieve higher levels of student success. Collaboration at all levels, if effectively promoted, holds significant potential for improving educational outcomes. Skvoretz et al. (2023) [33] reinforce this conclusion by highlighting the importance of integrating faculty social networks that connect research and teaching activities. They assert that aligning these networks can be instrumental in advancing instructional reforms in STEM education. Furthermore, collaboration between schools across countries in Latin America has been shown to foster the positive exchange of information and experiences, ultimately leading to improved school performance [14].

4.2 Conclusions

The greater the classroom interactions, the higher the grades one can expect from the students. The results indicate that the inside network statistics are positively correlated with students' grades, aligning with existing literature. In the classroom setting, the educational environment is guided by the teacher and supported by appropriate learning materials. However, a key difference in our findings compared to the literature is that approximately 90% of the students selected their co-work partners for classroom activities. In contrast, much of the literature highlights the teacher's central role in group selection and promoting participation. This observed factor might have influenced the statistical significance of the correlations. Future research could benefit from implementing control groups where teachers have a greater role in organizing work groups within the classroom. This approach could help disentangle the effects of teacher influence on group organization and its impact on grades.

Additionally, evidence suggests that some students act as bridges between different study groups, demonstrating a combination of acceptable grades and high sociability. These students could be a focal point for teacher-led interventions aimed at reaching more students and improving learning processes to enhance academic performance and could serve as a strategy for improving underperforming schools. Literature on international students and those with special requirements supports the idea that social network interventions can improve learning environments and promote inclusivity in classroom dynamics.

Conversely, the outside network was negatively correlated with students' grades. This highlights the influence of external, non-observable factors on outside-the-classroom interactions, such as family and community support factors. While the data collected does not provide sufficient insight into these complex relationships, this remains an important area for future research. Investigating the dynamics of outside learning groups and understanding the criteria students use to select study partners could provide valuable insights.

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Conflict of interest

The authors declare that there is no conflict of interest.

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