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RESEARCH ARTICLE

Monitoring Earth's ionosphere by means of hardware-software complex using the GPS/GLONASS satellite systems

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Abstract: Near real-time one-dimensional vertical electron density profiles are determined from GPS-derived total electron content (TEC) data by means of the iterative conjugate gradient projection method (CGP). Electron density profiles are determined in near realtime (within minutes of the time of measurement) from short time series of slant TEC (STEC) approximately 5 minutes. Measured STEC values are obtained from dual frequency data from a single GPS satellite at a single dual frequency receiver station. Both code-based TEC derived from the P-observable (Ptec) and phase-based TEC derived from the carrier phase observable (Ltec) are used in the solution. The CGP method addresses the ill-posed inverse problem of determining the electron density profiles from TEC measurements through the application of a side constraint to the acceptable solution. This is an iterative method which approximates the solution of a least squares problem through a converging sequence of solutions. The accuracy of the results is verified by comparison to electron density determined from the ionograms measured with Digisondes (Pushkov Institute of Terrestrial Magnetizm, Ionosphere and Radio Wave Propagation, Russian Academy of Science) located at Troizk, Moscow region (55.5N, 37.3E). The results of a hardware-software complex intended for monitoring the Earth's ionosphere according to navigation satellite systems are presented. The anomalous behavior of the critical frequency of the F2-layer ionosphere at latitudes 57-59 degrees observed in December 2014 is detected.

Keywords: electron density profile, conjugate gradient projection, ionosphere, international reference ionosphere

1 Introduction

Near real-time one dimensional vertical electron density profiles of the Earth's ionosphere are of interest in Space Physics for studying the dynamic behaviour of the ionosphere and for several practical applications including high frequency (HF) direction finding. GPS data from a single dual frequency receiver observing several GPS satellites provides a means of determining the near realtime total electron content (TEC) in a conic section of the ionosphere over the receiver station.

Computerised Ionospheric Tomography (CIT) is a method whereby the slant TEC (STEC) along a series of ray paths can be inverted to produce vertical electron density profiles. Several algorithms for CIT are documented in the literature[1,2]. Most algorithms require propagation delay data from several GPS or low earth orbit (LEO) satellites as observed from several ground-based receivers. The radiotranslucence method permits an inversion from data obtained from only one satellite as observed by one receiver.

The inversion of integral values to derive the underlying density profile constitutes an illposed inverse problem. Ill-posed inverse problems are characterized by strong dependence of the resulting electron density profile solution on errors in the operator matrix and on the initial STEC data used for the inversion. The solution depends discontinuously on the errors in the TEC data. Thus, the solution cannot be obtained by a simple matrix inversion. Some form of "regularization" of the solution is required, *i.e.* "regular" or acceptable solutions should somehow be favoured among the infinite number of possible solutions.

The results illustrated here allow near real-time CIT using data from only one satellite and one receiver, by means of the conjugate gradient projection (CGP) algorithm[3,4]. The CGP algorithm is based on an iterative

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conjugate gradient search with correction of the intermediate profiles using a priori data on electron density profiles. For solution an initial approximation of the desired electron density is derived from the International Reference Ionosphere (IRI) model of 2012. This method permits the derivation of electron density profiles in near real-time (with at most 1 minute delay from the time of measurement) from short timeseries of STEC values (5 minutes) using dual frequency data from a single satellite at a single GPS receiver station. The profiles can be determined at the sub-ionospheric points for each satellite visible from the GPS receiver along the whole trajectory of the sub-ionospheric point.

2 Integral equation formulation

The process of determining electron density profiles from STEC data requires the solution of an ill-posed inverse problem in the form of a Fredholm integral equation of the first kind *i.e.*

$$I\ (s)\ =\ \int_{P(s)} N_e(s)ds \tag{1}$$

where *I (s)* is the STEC along a ray path *P(s)* and *Ne (s)* is the electron density along the ray path.

The electron density distribution along the slant path *Ne (s)* is expressed in terms of a set of basis functions $f_j(s), j=1,\cdots,n$ through

$$N_e\ (s)\ =\ \sum_{j=1}^{n} \chi_j f_j(s) \tag{2}$$

where x_j are the unknown coefficients associated with each of the $j=1,\cdots,n$ basis functions. In general the basis function can be expressed as a product of functions which represent the horizontal, vertical and time variation of the electron density in the ionosphere *i.e.*

$$f_j(s) = f_j(\lambda, \psi, z, t) = H_j(\lambda, \psi)V_j(z)T_j(t) \tag{3}$$

where $H_j(\lambda,\ \psi)$ represents the horizontal (lon-lat) variation, V_j(z) represents the variation with altitude z, and T_j(t) represents the time variation.

In the CGP method, local spherical symmetry of the ionosphere is assumed over the volume spanned by the ray paths and the electron density profile is assumed to be constant for the duration of the measurement. In Equation 3 this assumption implies that over the sampled region $H_j(\lambda, \psi) = 1$.

Time invariance implies that for the duration of the measurement T_j(t)= 1. The vertical distribution of the electron density is assumed to be in the form of concentric fixed width uniform density layers, *i.e.*

$$V_j(z) = \begin{cases} 1,\ z_j,\ -\triangle z/2 < z < z_j\ +\ \triangle z/2; \\ \qquad\qquad z_{min} < z_j <\ z_{max} \\ 0,\ otherwise \end{cases} \tag{4}$$

The vertical range of the ionosphere is assumed to vary from Z_{min} (typically 80 km) to Z_{max} (typically 1000 km). With spherical symmetry the *i*-th ray path can be expressed as P_i(s) = $P(\theta\ _i$, Z), i = 1,$\cdots$, m, and the current density can be expressed as N_e(s) = N_e(z) with the zenith angle $\theta\ _i$ *i.e.* the angle between the ray path and the zenith direction at the receiver, and the altitude z above the receiver. Using these simplifications in the integral equation and setting f_j(s) = v_j(z) results in the discretized version of the integral Equation 1, becoming

$$\begin{aligned} b_i(\theta) &= \int_{P_i(s)} N_e(s)ds \\ &= \int_{P_i(\theta,\ z)} K(\theta,\ z)N_e(z)dz \\ &= \int_{P_i(\theta,\ z)} K(\theta,\ z) \sum_{j=1}^{n} \chi_j V_j(z)dz \\ &= \sum_{j=1}^{n} \chi_j \int_{P_i(\theta,\ z)} K(\theta,\ z)V_j(z)dz \\ &= \sum_{j=1}^{n} a_{ij}\chi_j \end{aligned} \tag{5}$$

where the *i*-index refers to the *i*-th ray path and a_{aj} the length of the *i*-th ray path through the *j*-th ionospheric layer, is given by

$$\begin{aligned} a_{ij} &= \int_{P_i(\theta,\ z)} K(\theta,\ z)V_j(z)dz \\ &= \int_{z_j-\Delta z/2}^{z_j+\Delta z/2} K(\theta_i,\ z)dz \\ &\approx \Delta z\ K(\theta_i,\ z_j) \end{aligned} \tag{6}$$

Equation 5 can be expressed as a matrix equation

$$A\chi = b \tag{7}$$

where A is the *m x n* operator matrix, x is the *n* x *1* vector of unknown layer densities, and b is the *m* x *1* set of measured STEC values.

The term $K(\theta$,z) is the Hilbert-Schmidt kernel of the integral equation given by

$$K(\theta,\ z)\ =\ \frac{ds(\theta)}{dz}\ =\ \left[1-\left(\frac{R_e}{R_e+z}\sin(\theta)\right)^2\right]^{-\frac{1}{2}} \tag{8}$$

Equation 8 defines the kernel K(θ, z) as the ratio between the slant path increment *ds* and the corresponding vertical increment *dz* for a ray path with zenith angle θ at altitude *z*. It is identical to the STEC to VTEC mapping function for an assumed ionospheric shell at altitude z.

The problem represented by the above matrix equation is an ill-posed problem since the matrix *A* is ill-conditioned. This means that the solution χ annot be determined by simple matrix inversion. Furthermore, matrix *A* is typically overdetermined (*m*>*n*) so that the solution can only be found in the least squares sense. The ill-conditioning is clear from the extreme ratio of its largest to smallest singular values as determined from the singular value decomposition of the matrix. The result of the ill-conditioning is that the solution χ is very sensitive to errors in *A* and *b*. Errors in b are mostly due to inaccuracies in the measurement of the slant TEC while errors in *A* are partly due to the discretization of the number of layers and partly due to the assumptions of spherical symmetry.

In the CGP algorithm the task of finding a solution is reduced to minimization of the functional

$$\phi(\chi,b)=\|A\chi-b\|^2 \tag{9}$$

This functional is defined for all functions that belong to a convex set *i.e.* which complies with the following restrictions:

$$M\downarrow_C=\begin{Bmatrix}\phi_\delta^{i+1}\ -\ \phi_\delta^i\ \leq 0,\ i=1,2,...,n\\ 0\leq\phi_\delta^i\leq C,\ i=2,3,...,n-1\end{Bmatrix} \quad (10a)$$

$$\overset{U}{M}\ =\begin{Bmatrix}\phi_\delta^i\ \geq\ 0,\ i=1,2,...,n\\ \phi_\delta^{i-1}-2\phi_\delta^i+\phi_\delta^{i+1}\leq 0,\ i=2,3,...,n-1\end{Bmatrix} \quad (10b)$$

$$\overset{U}{M}\ \downarrow\ =\begin{Bmatrix}\phi_\delta^i\ \geq\ 0,\ i=1,2,...,n\\ \phi_\delta^{i-1}-2\phi_\delta^i+\phi_\delta^{i+1}\leq 0,\ i=2,3,...,n-1\\ \phi_\delta^{i-1}-\phi_\delta^i\leq 0,\ i=2,3,...,n-1\end{Bmatrix} \quad (10c)$$

The CGP algorithm is a finite-dimensional difference equation approximation to the minimization of the above functional. The CGP algorithm is based on an iterative approach also because there is no necessity for a regularization parameter. It also differs through its implementation of an initial approximation χ_o, which should be monotonically varying, convex and non-negative in order to be an allowable electron density profile.

A uniform approximation to the exact solution can in principle be constructed, if the exact solution is a continuous function of the (limited) variation in the measured data. The structure of the electron density satisfies this restriction. Hence it is possible to construct an effective numerical algorithm for finding the electron density from measured STEC data. The use of a priori information allows one to search for the required solution by constraining acceptable solutions to the set of special functions with the properties of being monotonic, convex and positive definite.

Iterative methods do not solve for χ directly but determine the pseudo-inverse by first multiplying both sides of $A\chi$ = b with A^T which then produces the normal equation $A^T\ A\chi$ = A^Tb with a square coefficient matrix Q = A^TA . Here A^T indicates the transpose of *A*. Since the operator Ais linear and continuous, it makes ϕ (χ, b) a square-law function

$$\phi(x,b)=\langle x,Qx\rangle-2\langle Ax,b\rangle+\langle b,b\rangle \tag{11}$$

where x = N_e(z) is the vertical electron density solution, b(θ) is the slant TEC along the ray path with zenith angle θ and $<>$ denotes the scalar product of two *n x 1* vectors.

In the case of the CGP algorithm, minimization of the functional amounts to minimizing the discrepancy functional *i.e.* choosing the minimizing sequence for $\phi(\chi$,b) such that the residual norm of the solution, satisfies

$$\|Ax-b\|^2\ =\ \|e\|^2 \tag{12}$$

where *e* is the vector of discrepancies between the true STEC values and the measured STEC values along the finite set of chosen ray paths.

In functional analysis theory, it is shown that for convex set there is no need to find an exact minimum for the functional ϕ (x, b). It suffices to define an element of a sequence x_δ from a given set for which ϕ (x, b) $\leq \delta^2$, where δ is the measurement error. Thus to find an approximate solution we need to find a sequence of vectors $[x_\delta]_i$ associated with a sequence of values ${\phi_\delta}^i$ of the functional ϕ (x_δ) which minimizes on a convex set. When using a finite difference approximation of Equation 10, the sequence $[x_\delta]_j$ will become a set of solution vectors x_0, x_1,... x_i,...x_δ, the elements of which are such that ${\phi_\delta}^i$ will be subject to the restrictions of a convex set.

Iterative algorithms are widely used to solve inverse problems because they are very stable and allow one to halt the iteration procedure at any step, using given criteria. In the present case, the iteration is terminated when the discrepancy between the measured STEC and the STEC derived from the particular step in the solution reaches the value of the measurement error. The number of iterations to convergence is typically less than 100. The CGP method has some inherent regularization effect where the number of iterations plays the role of the regularization parameter.

The *i*-th step of the iterative CGP method to solve for x is given by

$$x_{i+1} = x_i \ - \ \alpha_i P_i \tag{13}$$

where

$$P_i \ = \ -grad\phi(x_i) + \beta_i P_{i-1} \tag{14}$$

and

$$\alpha_i \ = \ \frac{1}{2} \frac{\langle grad\phi(x_i), \ P_i \rangle}{\langle QP_{i,} \ P_i \rangle} \tag{15}$$

$$\beta_i \ = \ \frac{\| grad\phi(x_i) \|^2}{\| grad\phi(x_{i-1}) \|^2} \tag{16}$$

Note that gradϕ (x_i) denotes the gradient of the functional ϕ(x) evaluated at x = x_i. It can be shown that for ϕ(x) =$\|$Ax-b $\|^2$ the gradient is given by gradϕ(x)= $2A^T$ (Ax - b)

For the first step take P_0 = -gradϕ (x_o) where x_o is the initial approximation of the solution. The better the initial approximation, the faster the convergence. The IRI model proved to be a useful initial approximation in the CGP method. At each step in the inversion the parameters $(h_m F_2)_i$ and $N_m F_2)_i$ are derived from the electron density profile at x = x_i. If any element of the vector x = x_i becomes negative, the vector x = x_i is replaced with a new initial approximation x_0 which is taken from the IRI model for the electron density at the given time and location, adjusted by the estimate of $(h_m F_2)_i$ and $(N_m F_2)_i$ derived from the previous value of x_i. The above iterative calculation sequence is terminated when the norm of the residual becomes less than the measurement error. The CGP method may be more suitable to implement in hardware since it can be performed without actually calculating Q = $A^T A$ since $\langle$ Qp_i, p_i $\rangle$ = $\langle$ Ap_i, Ap_i $\rangle$. Hence the method requires less computer memory and fewer computations than the direct method.

The influence of the a priori information about the solution and of the error of the pseudorange measurements on the outcome of the tomographic inversion by the CGP method was investigated earlier using simulations[5]. It was shown that the application of the above inversion method for deriving the vertical electron density distribution is acceptably accurate if the error in the pseudorange measurements is not worse than 0.2 m. If this criterion is satisfied, the mean squared error in the derived vertical electron density distribution of the ionosphere as compared to the known solution used in the simulation does not exceed 0.02 NU (1 NU=10^6 electrons/cm^3), and the values of the maximum electronic density, NmF2, as derived from the inversion does not differ by more than 0.014 NU from the initially modeled value 0.82 NU.

Today the hardware and software complex of passive sounding the space environment based on the use of signals from navigation satellite systems has been created. In such a complex the continuous monitoring technology has been used. It is designed for the reconstruction of the spatiotemporal structure of the ionosphere and solving operational control tasks and full height distribution of the electron concentration of the ionosphere by radio translucence method on the track “satellite - the Earth” using radio navigation satellite system GPS/GLONASS in real time[6,7].

3 Hardware-software complex

A promising approach to ionosphere monitoring consists in the definition of the electron density profile in the altitude range of 100-1000 km, the parameters of the ionosphere layer F2 and TEC after processing of the received radio signals of global navigation satellite systems GLONASS and GPS. This approach is implemented in the proposed small-sized products HSC-MI (hardware-software complex of monitoring ionosphere) and it provides certain real-time distribution of the electron density at altitudes 100-1000 km along the trajectory of subionospheric points, the parameters of the ionosphere layer F2 and TEC in the passive (without emitting radio signals) mode in different azimuthal directions[7,8].

The structure of this product is shown in Figure 1; its functionalities are presented in Smirnov and Smirnova (2010)[8] and Smirnov (2007)[7]. It contains two parts: an antenna unit and an analog-to-digital module complex consisting of analog and digital parts of signals receiver. The output of the antenna unit is connected to the input module by a coaxial cable. The length of the antenna cable does not exceed 50 m. The receiver is connected to the computer by means of a combined USB/COM cable. Provided the electricity is available, this complex can be located anywhere.

The main element of hardware and software complex is a software module, based on the radio translucence method of the Earth’s ionosphere on the track “ground stationnavigation satellite”. Its practical realization is based on the use of the measurements of the radio signals parameters from the one receiver observations, an effective area of which covers more than 1000 km in radius[7].

By determining the electron density of the ionosphere over a specific territory it is possible to detect the pres-

Figure 1. Hardware-software complex for monitoring of the ionosphere: 1 - navigation receiver, 2 antenna, 3 - computer, 4 - power adapter, 5 - antenna cable, 6 - interface cable.

ence of potential specific features in its distribution. During the day, with one receiver we can get more than 100 trajectories of subionospheric points, each of them contains an average of about 20,000 measurements at 1 s steps. This allows us to get more than 20,000 vertical profiles of electron concentration, examples of which are shown in Figure 2.

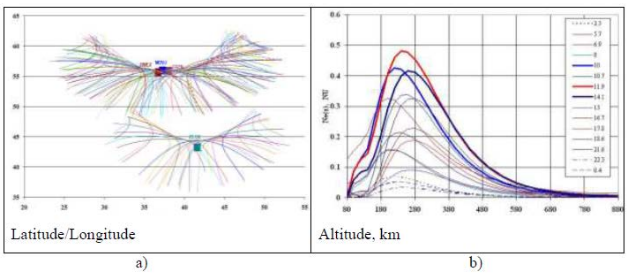

Figure 2. Trajectory of subionospheric points a) and electron density profiles b) obtained during the day on observations from one receiver

The hardware-software complex of monitoring ionosphere has been tested throughout the year. The complex is located in Troitsk, on the territory of the Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences in the immediate vicinity of the vertical sounding ionosonde DPS-4.

HSC-MI has a high degree of automation and provides a 24h continuous operation mode with thematic and service information archiving. An example of thematic mapping and service information on the screen HSC-MI is shown in Figure 3. Characters displayed as asterisks presented in Figure 3 show the coordinates of subionospheric points for which ionosphere parameters have been obtained at a specific time. For comparison we used the results obtained according to the navigation satellites coordinates of subionospheric points which were closest to the location of the ionosonde DPS-4. The results of the ionosonde and hardware-software complex are shown in Figure 4 for two months.

On average, for the June month the average daily value

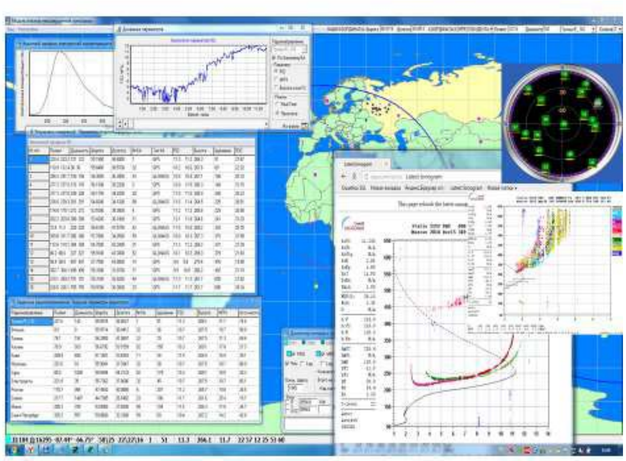

Figure 3. General view of the display of information from the AIC and ionosonde DPS-4 in terrestrial conditions

of RMS was 7% during the day - 6.65%, at night - 6.71%; for July month - 8.77%, 7.34% and 9.5%, respectively. Daily data take into account the results obtained for an hour before sunset and one hour after sunrise. Night data considers the results obtained one hour after sunset and an hour before sunrise. Daily data were obtained at the time of sunrise and sunset. HSC-MI was previously tested in sessions of short wave radio links[9].

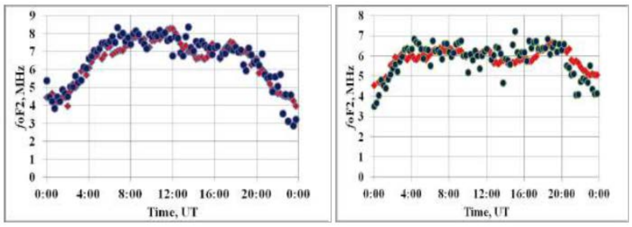

Figure 4. The values of the critical frequency of the ionosphere layer F2 for June 2 and July 2 (diamonds represent ionosonde data, circles represent HSC-MI data).

Some of the results obtained in December 2014 for several GLONASS and GPS satellites are presented in Figure 5. Here it shows the space-time variation of the electron concentration in the maximum of the F2-layer of the ionosphere, obtained from observations of the GPS satellite No. 2. The indicated satellite observed for several hours. The paths of the subionspheric points are shown in Figure 6. Figure 6 shows the detailed course of the critical frequency in the latitude range 57-59°, obtained from the observations of satellite No. 2 in the morning and evening hours of the day. The graphs clearly show a significant decrease in the critical frequency value in the indicated latitude range and the presence of a local maximum (indicated by an arrow). The maximum value in the region of decreasing corresponds approximately to the same latitude. This area was observed both in the morning and in the evening. The coordinates of the subionospheric point with a local maximum values of the critical frequency correspond to 45.03° E and 57.83° for the observation time of 3.98 hours and 28.39° E and 57.51°N for observation time of 16.45 hours. In Figure 6 these points are marked with an arrow. As can be seen, these points were observed at different azimuth directions relative to the location of the receiver (shown in Figure 6 with a diamond).

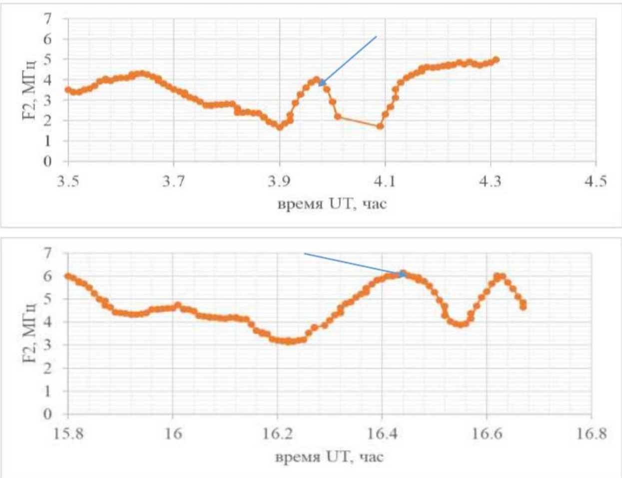

Figure 5. The change in the critical frequency of the F2-layer of the ionosphere along the trajectory of the subionospheric point.

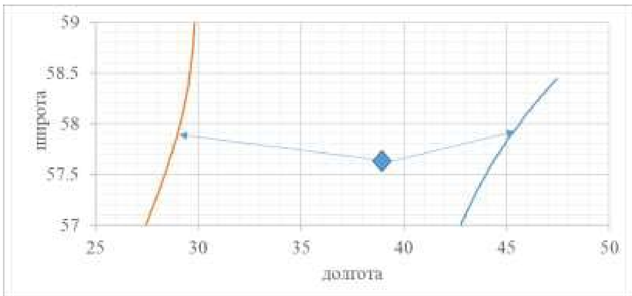

Figure 6. The trajectories of the subionospheric point in the period from 15 to 17 hours (left) and from 3.5 to 4.5 hours (right).

It should be noted that for the range of latitudes 57-59°, this course of variation of the critical frequency is similar for other observation sessions conducted during the day. At the same time, the latitude of the subionospheric point with a local maximum of the critical frequency is in the range of 57-59°, while the longitude of this point varies from 25 to 55°, depending on the observation time. The range of longitudes is limited by the scope of the hardware-software complex. It can be assumed that at these latitudes there is some kind of natural anomaly in the distribution of electron concentration.

4 Conclusions

The algorithm presented in this paper proves the feasibility of obtaining useful vertical electron density profiles in near real-time from GPS TEC data derived from a single GPS dual frequency receiver observing a single GPS satellite.

The algorithm presented here is able to reasonably well approximate the vertical structure of the ionosphere from short time-series of observations in near real time *i.e.* with only minutes delay between TEC measurement and derived profiles. The algorithms do not seem capable to resolve any detail of the E and F1 layers due to the lack of horizontal ray paths, which limit the vertical resolution of the electron density profile. It proved to be possible to determine simultaneously the vertical electron density profiles from most visible satellites with monotonically increasing or decreasing levations during the period of observation.

The vertical electron density profile is reasonably well approximated by the CGP method with the use of 40 slant TEC measurements over a period of 5 minutes. The vertical electron density profiles can be determined along the whole trajectory of the ionospheric pierce points of the satellite to receiver ray paths.

The accuracy of the electron density profiles obtained by CGP method from actual GPS data was verified by comparison with scaled ionograms from an ionosonde co-located with the GPS receiver. The hardware-software complex of monitoring ionosphere has been tested throughout the year. Using hardware-software complex intended for monitoring the Earth's ionosphere according to navigation satellite systems allowed to determine the anomalous behavior of the critical frequency of the F2 layer ionosphere observed at latitudes 57-59 degrees. Detection of such changes over a wide area became possible only when using satellite systems.

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