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RESEARCH ARTICLE

A Fuzzification Measure of Robust Design in Condition of "Desired Target Being Best" in Design

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Abstract: In the present article, a fuzzification measure of robust design in condition of "desired target being best" is regulated, which consists of the "complement" of the membership value of objective response and PMOO. The mean value of "complement" of the membership value of a set of test data of objective response belonging to its desired target value in fuzzification is taken as an indicator to join the assessment of the 1st part of partial preferable probability of the objective; the dispersion of a set of test data in term of membership with regard to the desired target value is taken as the other indicator to participate the assessment of the 2nd part of partial preferable probability of the objective. Moreover, the fuzzification measure of robust design is regulated in term of PMOO. As utilizations, two instances are presented to illuminate the regulation in design.

Keywords: fuzzification, membership value, robust design, target being best

1 Introduction

As to multi-objective optimization (MOO), an inexact or linguistic description for responses appears in some cases, which leads to the assessments with characteristic of "fuzzy" in some sense [1–5], such problem has been primarily solved in recent research with the fuzzed PMOO (probabilistic multi - objective optimization approach) [6–8].

Subsequently, a fuzzification measurement is put forward to deal with the MOO problem for the problem of "desired target being best" flexibly [8]. The closeness degree of the experimental data to its desired target value of an attribute is characterized by the "membership of the data belonging to the desired target value", and the membership value is directly used as the utility of the objective to join the assessment of PMOO. Furthermore, the membership u was used as the beneficial indicator, *i.e.*, "the larger the better" type, to conduct the PMOO evaluation [8].

However, since the maximum value of membership u is 1 exclusively, which is a finite value, instead of infinite; so an appropriate manner to deal with this problem is needed. Additionally, in condition of robust assessment, the spreading of experimental data must be taken into account in proper manner as well.

In this article, an alternative regulation is put forward by introducing the "complement" of the membership value, *i.e.*, $\eta = 1 - u$ as an indicator logically to deal with the matter [3], which forms a rational fuzzification regulation of robust design in term of PMOO in condition of "desired target being best"; moreover, two instances are represented to illuminate the regulation.

2 Rational Fuzzification Regulation of Robust Design in Condition of "Desired Target Being Best" in Term of PMOO

2.1 Membership Value and Its Complement of an Objective in Condition of "Desired Target Being Best"

Above discussion indicates that the membership value and its complement of an attribute in condition of "desired target being best" can be introduced to characterize the closeness degree of the test data to its desired target value [6–8], the corresponding algorithm can be performed as following,

$$\begin{cases} u(f) = 1,\, f = f_0 \\ u(f) = 1 - \frac{(|f - f_0|)}{\delta},\, |f - f_0| \leq \delta \\ u(f) = 0,\, |f - f_0| > \delta \end{cases} \tag{1}$$

In Eq. (1), $u(f)$ expresses the membership of experimental data f belonging the desired target value f_0 of the attribute; δ is the pre-assigned data for the critical value of distance of f from f_0, at which the value of $u(f)$ decreases to 0.

As to the condition of "desired target being best", since the limit value of membership $u(f)$ of an attribute response f belonging to f_0 is "1" only, *i.e.*, a finite value instead of infinitely large one, which is not exactly consistent with the essence of "the larger the better" type of index. So, it seems improper to take membership $u(f)$ as a beneficial indicator to conduct this optimization problem directly, since in the latter case the value of the attribute response has the possibility to get a value of infinitely large instead of finite one.

Alternatively, a flexible measure could be introduced to use the "complement" η of the membership value $u(f)$ as an indicator to perform the optimization. The definition of the "complement" η of the membership value $u(f)$ is shown by Eq. (2).

$$\eta = 1 - u \tag{2}$$

Obviously, the lower limit value of η is 0, which corresponds to u taking its maximum value of 1. Therefore, the optimization problem of u approaching its maximum value is equivalent to η inclining to its minimum value of 0.

Furthermore, as to robustness assessment, since the inevitability of spreading of a set of test data at the same experimental conditions due to the effects of external uncertain factors, the evaluation of scattering of a set of test data must be taken into account surely [6–8].

In the light of Lin and Tu's discussion [9], the scattering of a set of test data in term of membership of fuzzy theory can be characterized by Eq. (3).

$$s_{\mathrm{u}} = (\bar{\eta}^2 + \sigma_{\mathrm{u}}^2)^{0.5} \tag{3}$$

In Eq. (3), σ_u indicates the standard deviation of membership value u of a set of test data at the same experimental conditions; $\bar{\eta}$is the mean value of "complement" η of the membership value u in the corresponding set, which is an unbeneficial index to join the assessment of the 1^{st} part of partial preferable probability; s_u is in fact the indicator of scattering of a set of test data in term of membership with regard to the desired target value to participate the assessment of the other part of partial preferable probability.

2.2 Assessment of Preferable Probability

Furthermore, the assessment of two parts $P_{\bar{\eta}}$ and $P_{s_{\mathrm{u}}}$ of partial preferable probability can be done by taking both $\bar{\eta}$and s_u of an attribute as unbeneficial type of dual indexes [6–8]. As a result, the partial preferable probability P_{kl} is the product of both two parts $P_{\bar{\eta}}$ and $P_{s_{\mathrm{u}}}$of an attribute.

Subsequently, the overall preferable probability P_k of k^{th} alternative candidate is the product of its all partial preferable probability P_{kl} [6–8].

$$P_k = \prod_{l=1}^{b} P_{kl}, k = 1, 2, ..., a; l = 1, 2, ..., b \tag{4}$$

Finally, the optimal option is the specific alternative candidate that has the largest overall preferable probability.

3 Utilization Examples for Illustration

3.1 Parameter Design of Leaf Spring with Targeted Free Height of 7.6 Inches

Montgomery mentioned the parameter design of leaf spring problem [10], which was once originally discussed by Pignatiello Jr. et al. [11]. Their article studied the application of the parametric effect of five input variables on the free height of truck leaf springs. The parameters included: furnace temperature - I_1; heating time - I_2; transfer time - I_3; hold down time - I_3, and quench oil temperature - I_4. Especially, the quench oil temperature was taken as the noise variable.

Here it is restudied by using fuzzification regulation. The experimental result data are cited in Table 1 [10]. The optimal design aims to option parameters so as to ensure the desired target value of the free height around f_0 = 7.6 inches with possible smaller spreading [10].

Table 1 Experimental results of leaf spring free height

No.	Input parameter				Value of free height in two noise levels, f (Inch)					
	I_1	I_2	I_3	I_4	I_{5+}			I_{5-}		
1	-	-	-	-	7.50	7.25	7.12	7.78	7.78	7.81
2	+	-	-	+	7.88	7.88	7.44	8.15	8.18	7.88
3	-	+	-	+	7.50	7.56	7.50	7.50	7.56	7.50
4	**+**	**+**	**-**	**-**	**7.63**	**7.75**	**7.56**	**7.59**	**7.56**	**7.75**
5	-	-	+	+	7.32	7.44	7.44	7.54	8.00	7.88
6	+	-	+	-	7.56	7.69	7.62	7.69	8.09	8.06
7	-	+	+	-	7.18	7.18	7.25	7.56	7.52	7.44
8	+	+	+	+	7.81	7.50	7.59	7.56	7.81	7.69

As to fuzzification assessment, the membership value u of a free height f belonging to its desired target value f_0 = 7.6 inches needs to be conducted by employing Eq. (1) first in principle. In the assessment, if a pre-assign data of δ is given, for example δ = 0.6 inches, then it derives the evaluation expression of membership belonging to its desired target value of 7.6 inches for this problem according to Eq. (5).

$$\begin{cases} u(f) = 1,\ f = 7.6; \\ u(f) = 1 - \frac{(|f-7.6|)}{0.6},\ |f-7.6| \leq 0.6 \\ u(f) = 0,\ |f-7.6| > 0.6 \end{cases} \tag{5}$$

Consequently, Table 2 represents the membership values u and the corresponding errors of the free height values shown in Table 1.

Table 2 Membership function u and errors of each tested free height

No.	Membership function u								
	I_{5+}			I_{5-}			$\bar{u}$	σ_u	s_u
1	0.8333	0.4167	0.2000	0.7000	0.7000	0.6500	0.5833	0.2117	0.4674
2	0.5333	0.5333	0.7333	0.0833	0.0333	0.5333	0.4083	0.2578	0.6454
3	0.8333	0.9333	0.8333	0.8333	0.9333	0.8333	0.8667	0.0471	0.1414
4	**0.9500**	**0.7500**	**0.9333**	**0.9833**	**0.9333**	**0.7500**	**0.8833**	**0.0957**	**0.1509**
5	0.5333	0.7333	0.7333	0.9000	0.3333	0.5333	0.6278	0.1830	0.4148
6	0.9333	0.8500	0.9667	0.8500	0.1833	0.2333	0.6694	0.3291	0.4664
7	0.3000	0.3000	0.4167	0.9333	0.8667	0.7333	0.5917	0.2624	0.4854
8	0.6500	0.8333	0.9833	0.9333	0.6500	0.8500	0.8167	0.1280	0.2236

Furthermore, the evaluation results for preferable probability are conducted and presented in Table 3, which indicates the alternative candidate No. 4 giving the largest overall preferable probability, therefore optimum option of this optimal problem is the alternative candidate No. 4.

Table 3 Assessment results of preferable probability

No.	$\bar{\eta} = 1 - \bar{\mu}$	s_μ	$P_{\bar{\eta}}$	P_{s_μ}	$P_t \times 10^2$	Rank
1	0.4167	0.4674	0.0937	0.0968	0.9069	6
2	0.5917	0.6454	0.0375	0.0429	0.1606	8
3	0.1333	0.1414	0.1847	0.1956	3.6122	2
4	**0.1167**	**0.1509**	**0.1900**	**0.1927**	**3.6622**	**1**
5	0.3722	0.4148	0.1079	0.1128	1.2172	4
6	0.3306	0.4664	0.1213	0.0971	1.1782	5
7	0.4083	0.4854	0.0963	0.0914	0.8802	7
8	0.1833	0.2236	0.1686	0.1707	2.8781	3

3.2 Robust Design of a Clamping Mechanism Stroking under Orthogonal Experimental Condition

Robust design of a clamping mechanism stroking under orthogonal experimental condition was investigated by Wu et al. [12], the controllable input parameters include, x_1, x_2 and x_3; while the machining errors of x_1, x_2 and x_3 are taken as the noise variables; the stroking's movement region f is the optimal attribute with robustness around its desired target value f_0 of 525.00 mm.

Table 4 cites the data of the design of controllable input parameters and machining errors. Table 5 cites the simulated consequences by using ADAMS technique. The designs $L_4(2^3)$ and $L_9(3^4)$ were used for outer table and inner table of orthogonal experimental condition in Wu's study, individually.

Table 4 Designed levels of input parameters

Level	Controllable variable			Noise variable		
	x_1/mm	x_2/mm	x_3/°	Δx_1/mm	Δx_2/mm	Δx_3/°
1	369	300	95	–0.02	–0.02	–0.10
2	379	311	98	0.02	0.02	0.10
3	389	320	100			

Table 5 Simulated results with $L_4(2^3)$ and $L_9(3^4)$ for outer and inner variables

Variable	Inner table $L_9(3^4)$				Outer table $L_4(2^3)$				No./Variable
					1	2	3	4	
No.	1	2	3	4	1	1	2	2	Δx_1
					1	2	1	2	Δx_2
	x_1	x_2	x_3	ex	1	2	2	1	Δx_3
No.					Consequence, f / mm				
1	1	1	3	1	507.554	508.469	508.313	507.652	
2	**1**	**2**	**2**	**2**	523.847	524.858	**524.688**	**523.947**	
3	1	3	1	3	534.906	536.03	535.845	535.007	
4	2	1	2	3	488.239	489.234	489.087	488.334	
5	2	2	1	1	501.651	502.747	502.589	501.747	
6	2	3	3	2	552.237	553.185	553.015	552.338	
7	3	1	1	2	468.327	469.392	469.253	468.419	
8	3	2	3	3	521.454	522.404	522.253	521.552	
9	3	3	2	1	531.098	532.139	531.979	531.169	

Table 6 presents the membership values u of the stroking movement region f belonging to its desired target value f_0 = 525 mm in case of δ = 57 mm, and their mean value $\bar{u}$.

Table 6 Membership values μ and its mean value $\bar{\mu}$

No.	μ				$\bar{\mu}$
1	0.6939	0.7100	0.7072	0.6956	0.7017
2	0.9798	0.9975	0.9945	0.9815	**0.9883**
3	0.8262	0.8065	0.8097	0.8244	0.8167
4	0.3551	0.3725	0.3699	0.3567	0.3636
5	0.5904	0.6096	0.6068	0.5921	0.5997
6	0.5222	0.5055	0.5085	0.5204	0.5141
7	0.0057	0.0244	0.0220	0.0074	0.0149
8	0.9378	0.9545	0.9518	0.9395	0.9459
9	0.8930	0.8748	0.8776	0.8918	0.8843

Table 7 shows the evaluated results of η, s and values of partial and overall preferable probabilities, which reflect that the alternative candidate No. 2 exhibiting largest overall preferable probability, therefore alternative candidate No. 2 can be the primary selection of this robust design.

Table 8 is the range analysis of this assessment by means of overall preferable probability. The consequences in Table 8 reflect the optimal configuration bing $x_1 1$, $x_2 2$, $x_3 2$, it is exactly the alternative candidate No. 2., and impact order of input variables is $x_2 > x_1 > x_3$.

Table 7 Evaluated results of η, s and partial and overall preferable probabilities

No.	σ_u	$\bar{\eta} = 1 - \bar{u}$	S_u	$P_{\bar{\eta}}$	P_{s_u}	$P_t \times 10^2$	Rank
1	0.0070	0.2983	0.2984	0.1204	0.1205	1.4505	5
2	**0.0078**	**0.0117**	**0.0140**	**0.1698**	**0.1693**	**2.8758**	**1**
3	0.0087	0.1833	0.1835	0.1403	0.1402	1.9663	4
4	0.0077	0.6364	0.6365	0.0621	0.0623	0.3873	8
5	0.0086	0.4003	0.4004	0.1028	0.1029	1.0584	6
6	0.0072	0.4859	0.4859	0.0881	0.0882	0.7771	7
7	0.0084	0.9851	0.9852	0.0020	0.0024	0.0005	9
8	0.0073	0.0541	0.0546	0.1625	0.1623	2.6385	2
9	0.0082	0.1157	0.1160	0.1519	0.1518	2.3057	3

Table 8 Range analysis of the total preferable probability

Level	x_1	x_2	x_3
1	2.0975	0.6128	1.008
2	**0.7409**	**2.1909**	**1.986**
3	1.6482	1.6830	1.553
Range	1.3566	1.5781	0.9778
Impact	2	1	3
Optimal conf.	1	2	2

4 Conclusion

This study indicates that the combination of PMOO with fuzzification is effective; the introduction of "complement" of the membership value is a proper indicator to perform the assessment of robust design in condition of "desired target being best"; all these procedures consist of the regulation of fuzzification measure reasonably.

Conflicts of interest

The authors declare that they have no conflict of interest.

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